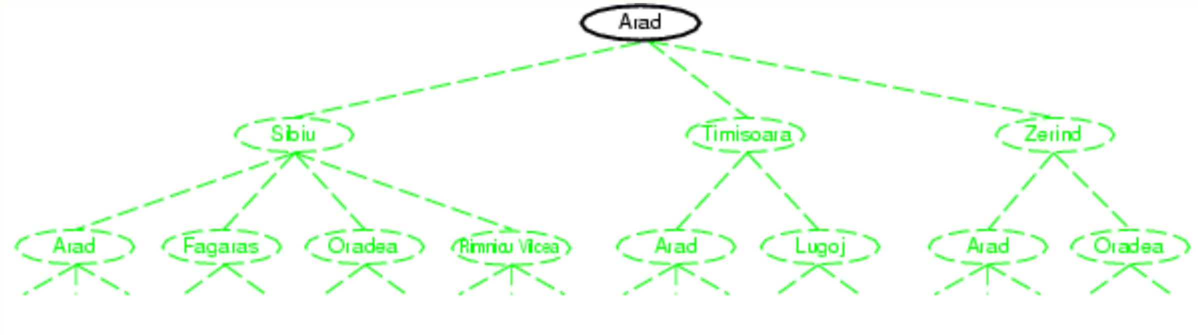
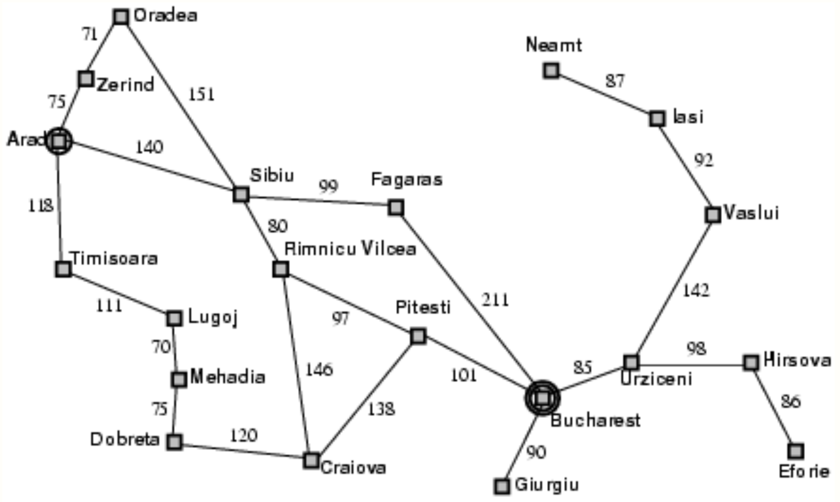
A* Search

Tree search algorithms

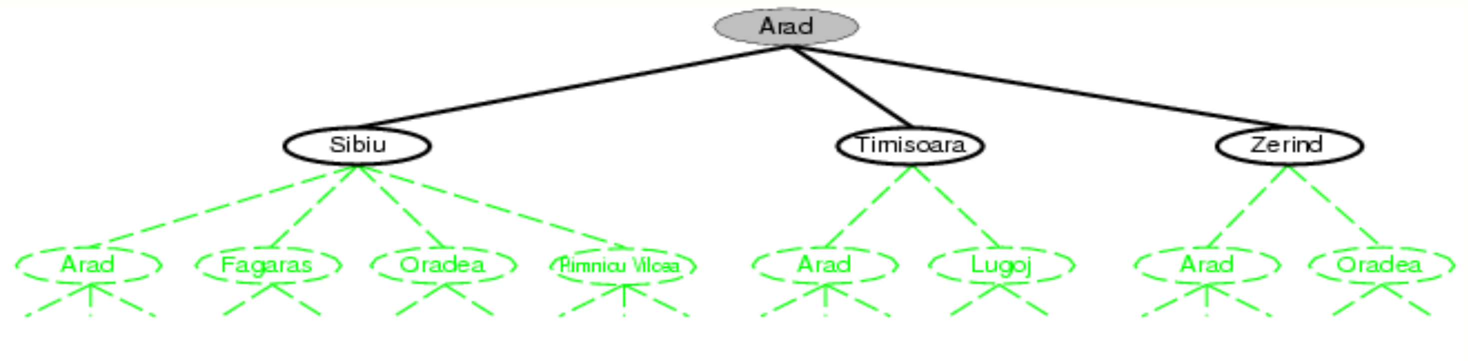
- Basic idea:
  - Exploration of state space by generating successors of already-explored states (a.k.a.~expanding states).
  - Every states is evaluated: *is it a goal state*?

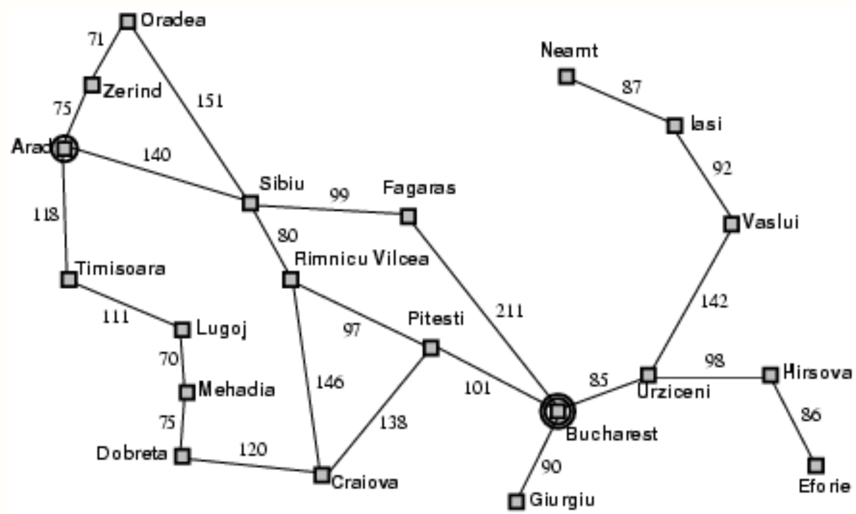
Tree search example





Tree search example





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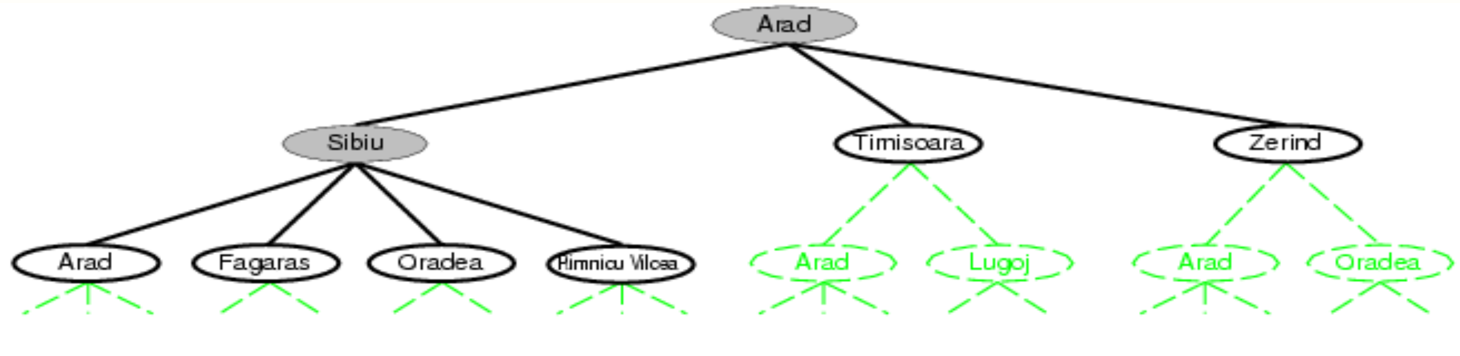


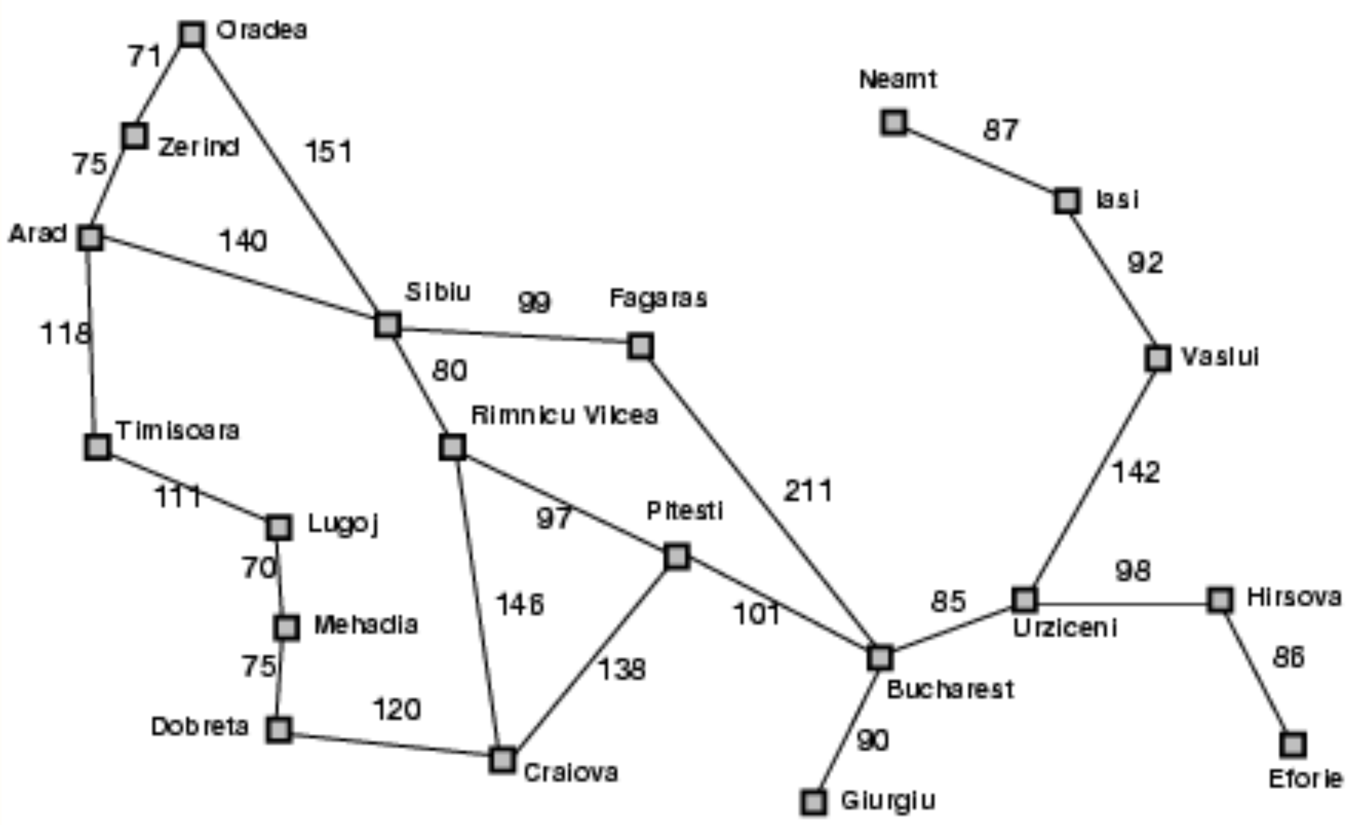
function TREE-SEARCH(*problem*, *strategy*) **returns** a solution, or failure
  initialize the search tree using the initial state of *problem*
  loop do
    if there are no candidates for expansion **then return** failure
    choose a leaf node for expansion according to *strategy*
    if the node contains a goal state **then return** the corresponding solution
    else expand the node and add the resulting nodes to the search tree

Best-first search

- Idea: use an evaluation function *f(n)* for each node
  - f(n) provides an estimate for the total cost.
  - → Expand the node n with smallest f(n).

- Implementation:
  Order the nodes in fringe increasing order of cost.

Romania with straight-line dist.



Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
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Hirsova	151
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A^* search

- Idea: avoid expanding paths that are already expensive
- Evaluation function $f(n) = g(n) + h(n)$
- $g(n)$ = cost so far to reach n
- $h(n)$ = estimated cost from n to goal
- $f(n)$ = estimated total cost of path through n to goal
- Best First search has $f(n)=h(n)$
- Uniform Cost search has $f(n)=g(n)$

Admissible heuristics

- A heuristic $h(n)$ is admissible if for every node n, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)
- Theorem: If $h(n)$ is admissible, A^* using `TREE-SEARCH` is optimal

Dominance

- If $h_2(n) \geq h_1(n)$ for all n (both admissible)
- then h_2 dominates h_1
- h_2 is better for search: it is guaranteed to expand less or equal nr of nodes.

- Typical search costs (average number of nodes expanded):

- *d=12* IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- *d=24* IDS = too many nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes

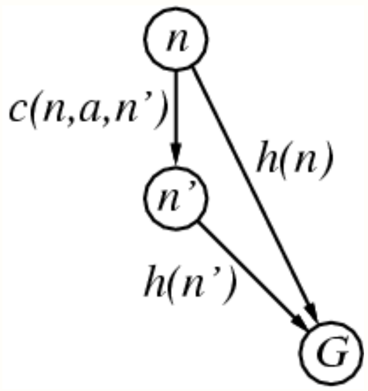
Relaxed problems

- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution

Consistent heuristics

- A heuristic is consistent if for every node *n*, every successor *n'* of *n* generated by any action *a*,

$$h(n) \le c(n,a,n') + h(n')$$



- If *h* is consistent, we have

$$\begin{aligned}
f(n') &= g(n') + h(n') && \text{(by def.)} \\
&= g(n) + c(n,a,n') + h(n') && (g(n')=g(n)+c(n.a.n')) \\
&\ge g(n) + h(n) = f(n) && \text{(consistency)} \\
f(n') &\ge f(n)
\end{aligned}$$

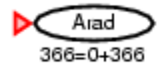
It's the triangle inequality !

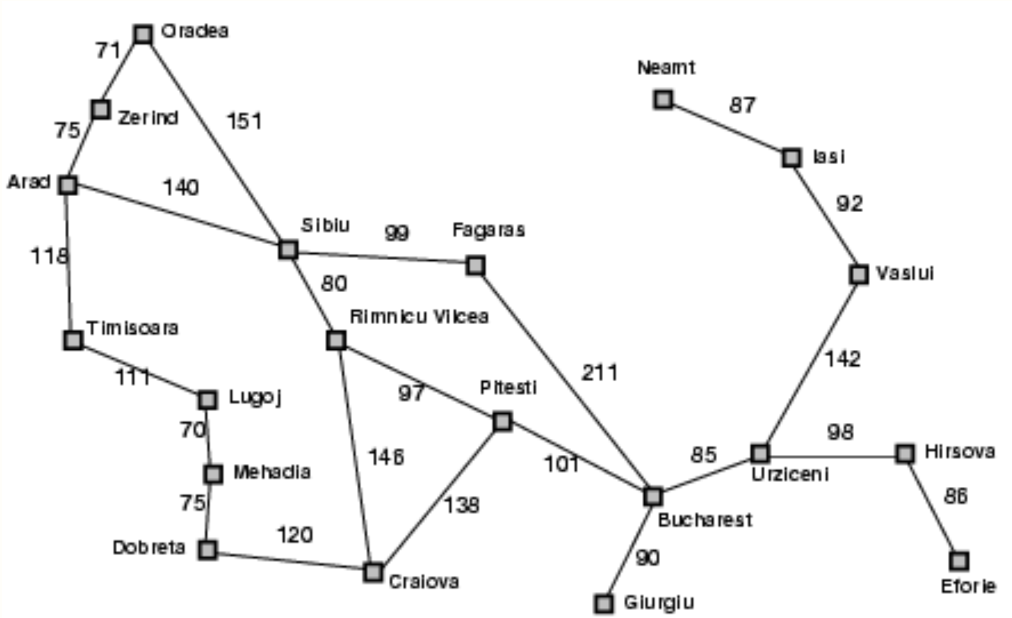
- i.e., *f(n)* is non-decreasing along any path.

keeps all checked nodes in memory to avoid repeated states

- Theorem:
  If *h(n)* is consistent, A* using GRAPH-SEARCH is optimal

A* search example





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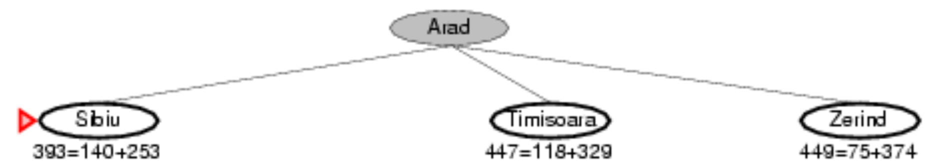

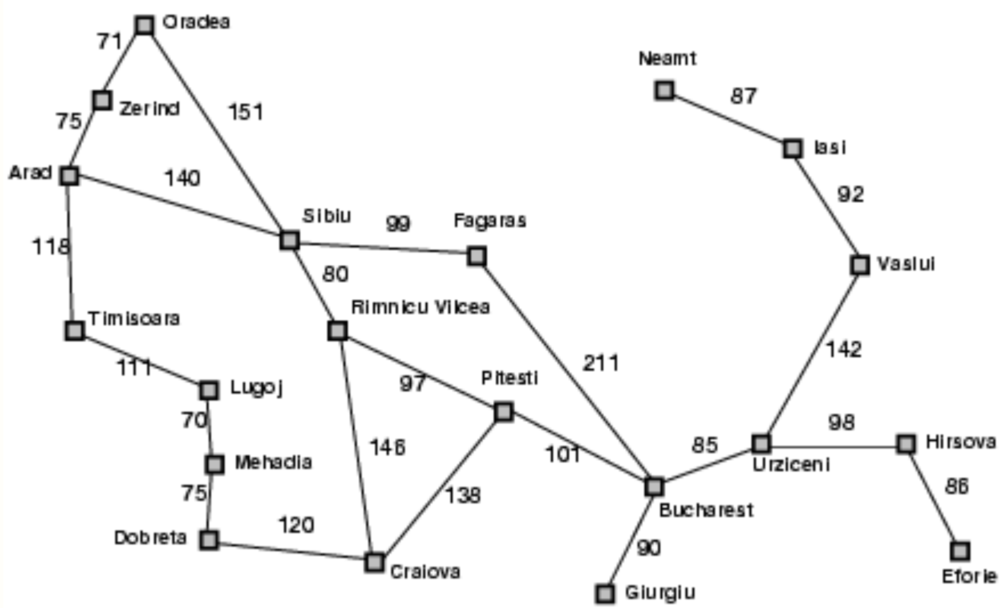

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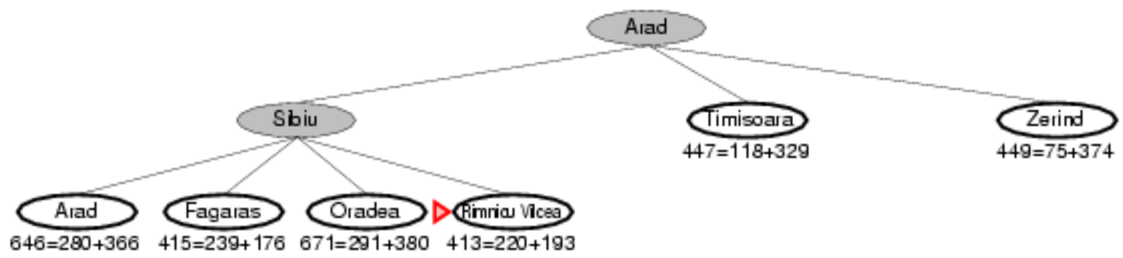

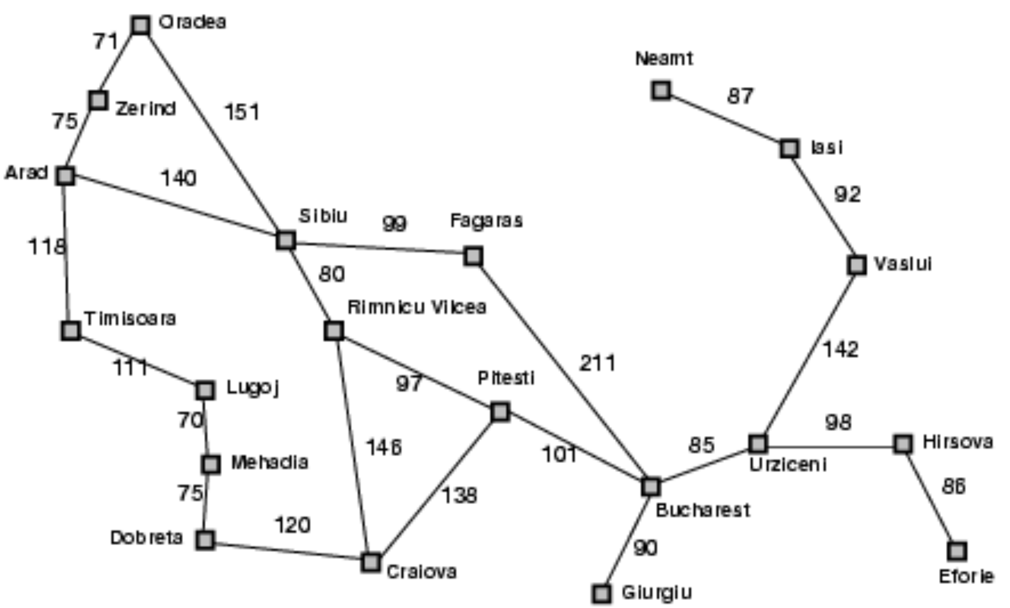

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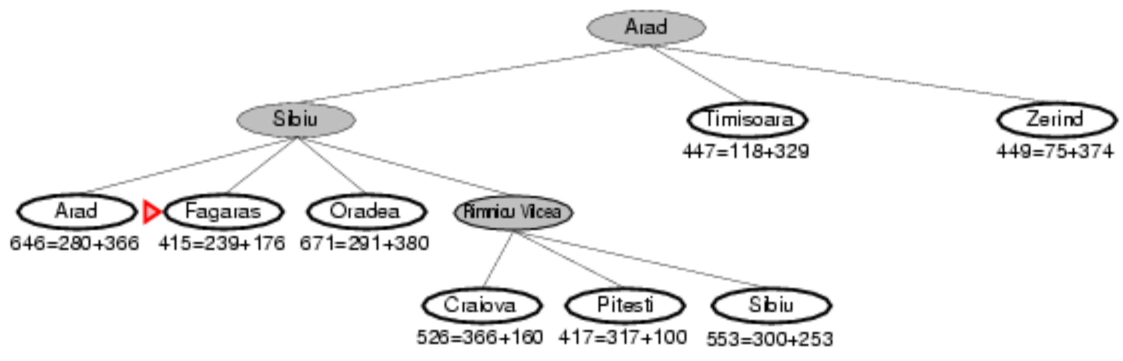

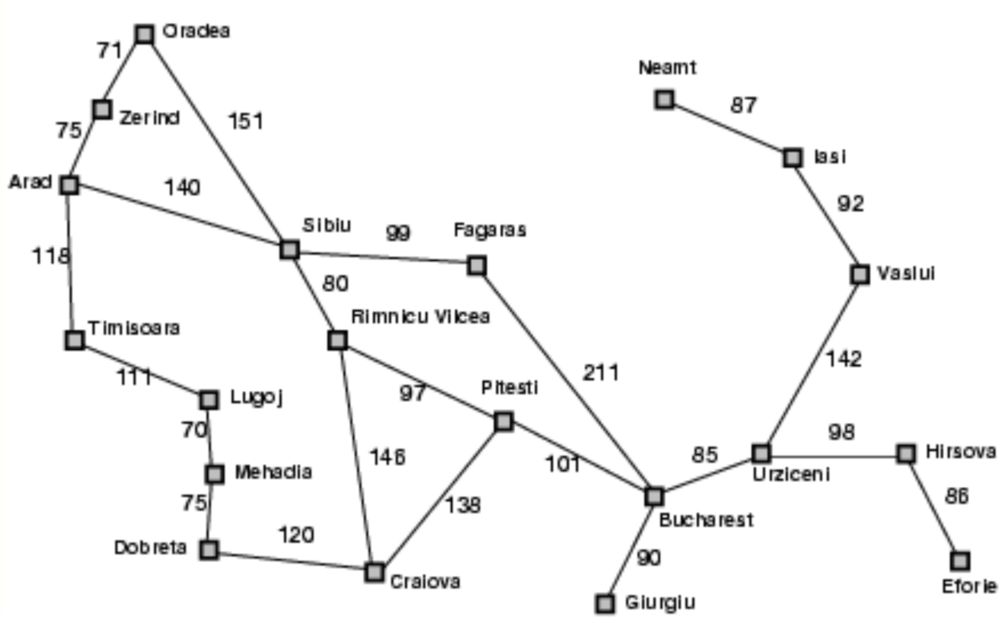

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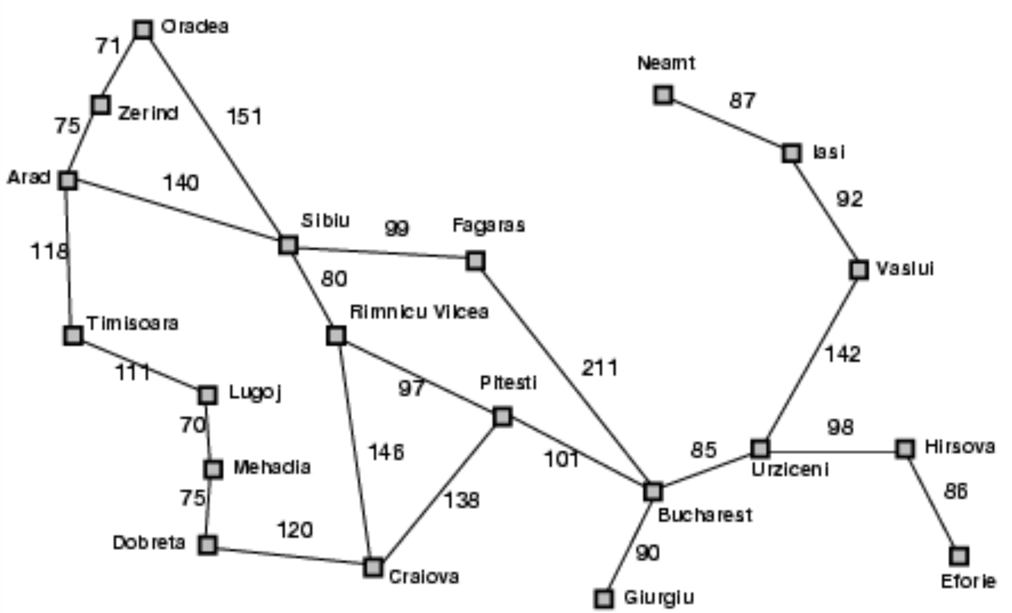

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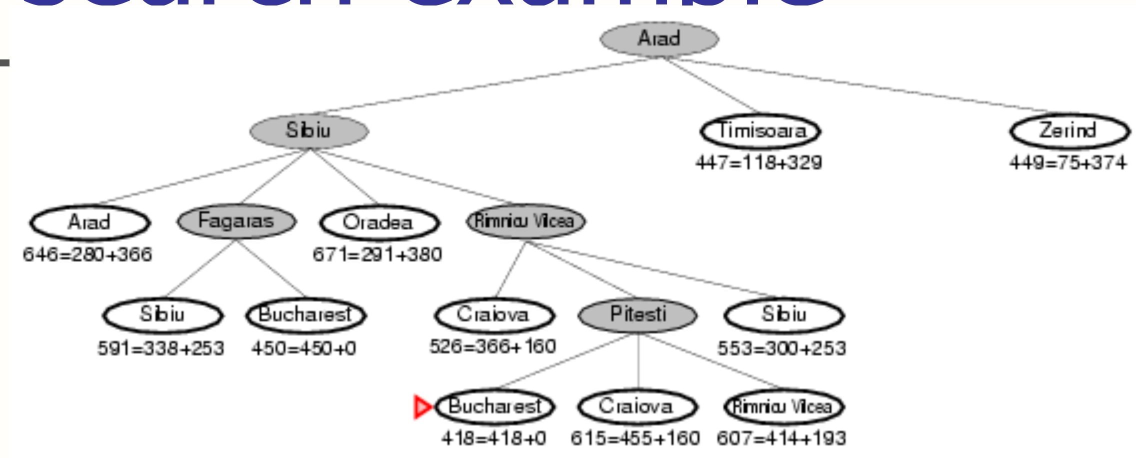
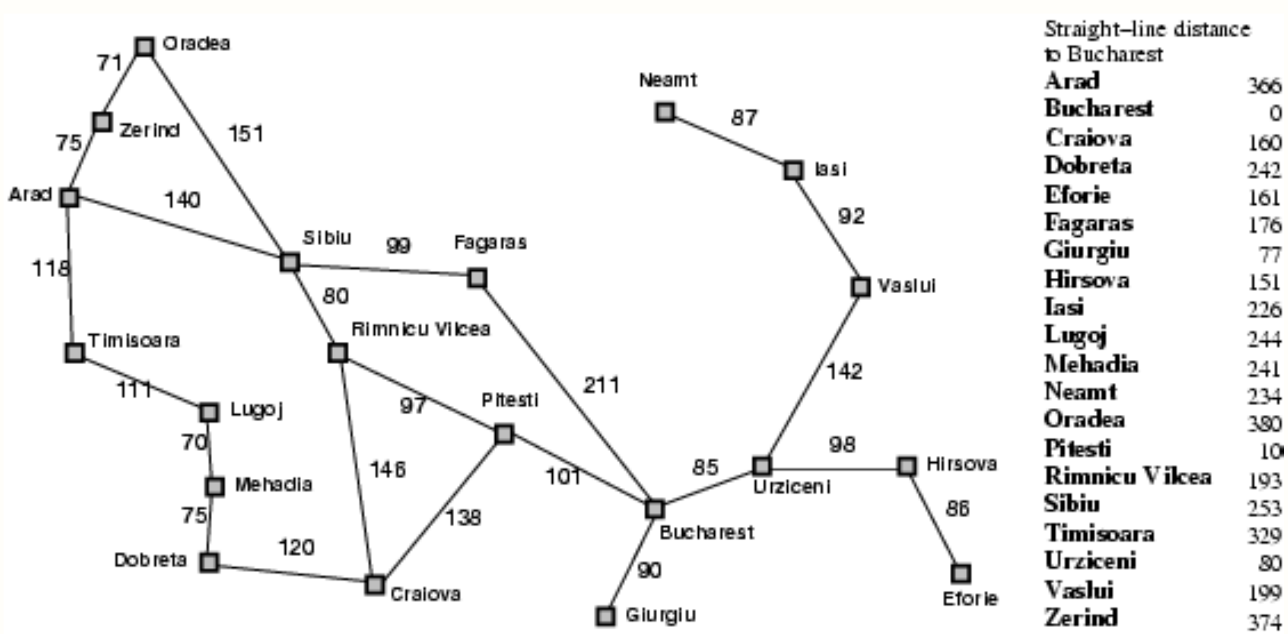




Properties of A*

- Complete? Yes (unless there are infinitely many nodes with f $\leq f(G)$, i.e. step-cost > ε)
- Time/Space? Exponential: b^d

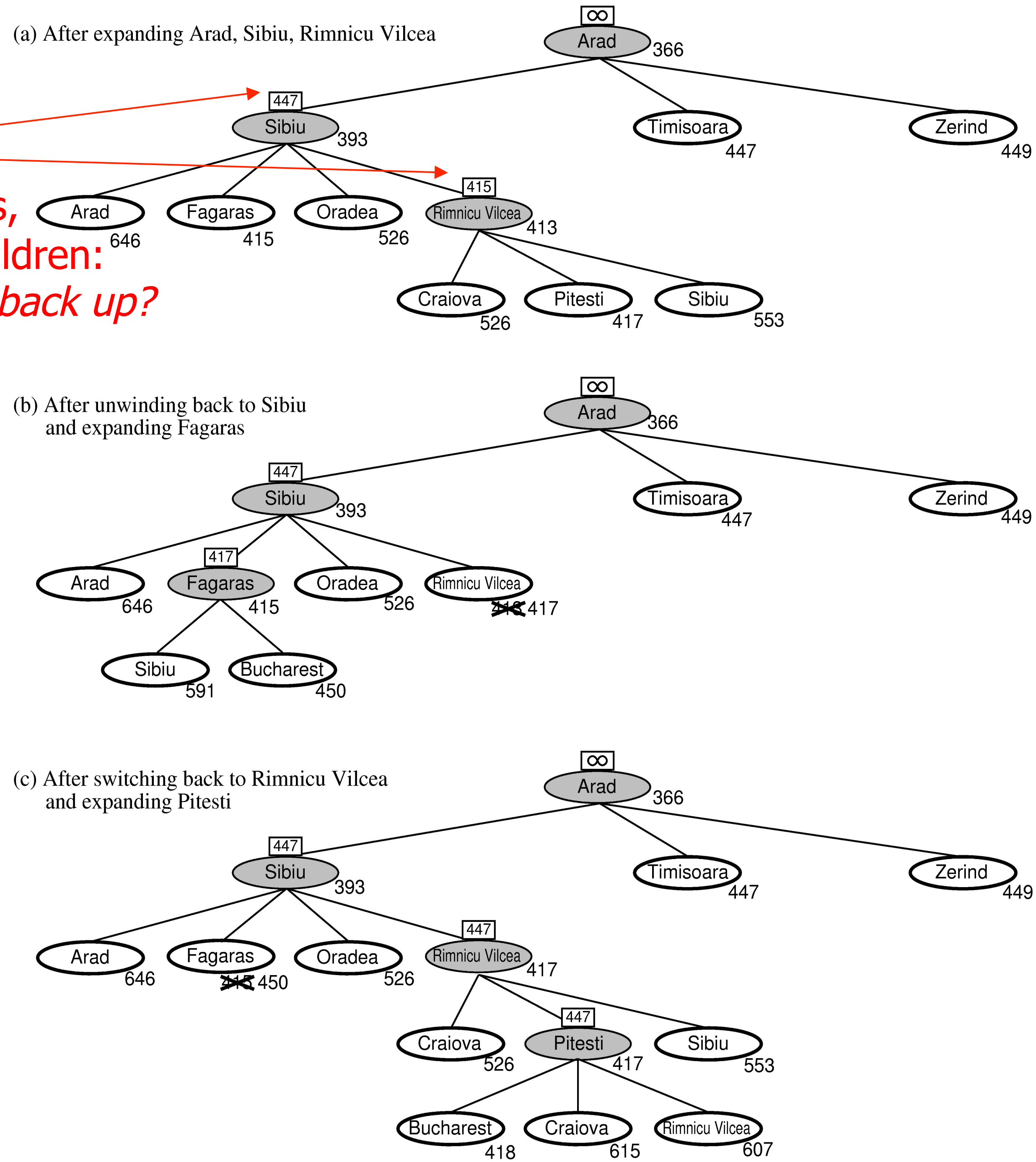
  except if: $|h(n) - h^*(n)| \leq O(\log h^*(n))$
- Optimal? Yes
- *Optimally Efficient*: Yes (no algorithm with the same heuristic is guaranteed to expand fewer nodes)

Memory Bounded Heuristic Search: Recursive BFS

- How can we solve the memory problem for A* search?
- Idea: Try something like depth first search, but let's not forget everything about the branches we have partially explored.
- *We remember the best f-value we have found so far in the branch we are deleting*.

RBFS:

best alternative
over fringe nodes,
which are not children:
i.e. do I want to back up?



RBFS changes its mind very often in practice.

This is because the f=g+h become more accurate (less optimistic) as we approach the goal. Hence, higher level nodes have smaller f-values and will be explored first.

Problem: We should keep in memory whatever we can.

Simple Memory Bounded A*

- This is like A*, but when memory is full we delete the worst node (largest f-value).
- Like RBFS, we remember the best descendent in the branch we delete.
- If there is a tie (equal f-values) we delete the oldest nodes first.
- simple-MBA* finds the optimal *reachable* solution given the memory constraint.
- Time can still be exponential.

A Solution is not reachable
if a single path from root to goal
does not fit into memory