
Figure 2.1 A divide-and-conquer algorithm for integer multiplication.

`function multiply(x, y)`

`Input: Positive integers x and y , in binary`

`Output: Their product`

`$n = \max(\text{size of } x, \text{ size of } y)$`

`if $n = 1$: return xy`

`$x_L, x_R = \text{leftmost } \lceil n/2 \rceil, \text{ rightmost } \lfloor n/2 \rfloor \text{ bits of } x$`

`$y_L, y_R = \text{leftmost } \lceil n/2 \rceil, \text{ rightmost } \lfloor n/2 \rfloor \text{ bits of } y$`

`$P_1 = \text{multiply}(x_L, y_L)$`

`$P_2 = \text{multiply}(x_R, y_R)$`

`$P_3 = \text{multiply}(x_L + x_R, y_L + y_R)$`

`return $P_1 \times 2^n + (P_3 - P_1 - P_2) \times 2^{n/2} + P_2$`

Figure 1.4 Modular exponentiation.

function modexp(x, y, N)

Input: Two n -bit integers x and N , an integer exponent y

Output: $x^y \bmod N$

if $y = 0$: **return** 1

$z = \text{modexp}(x, \lfloor y/2 \rfloor, N)$

if y is even:

return $z^2 \bmod N$

else:

return $x \cdot z^2 \bmod N$
