CISC 1100 Structures of Computer Science
Spring 2011 Practice Quiz SOLUTIONS

Name: _____

This exam is a closed book, closed notes, closed calculator, and closed computer, exam. You will not need to simplify your answers to a final numerical solution unless specifically directed to do so. Also, you may leave your answers with P(X,Y) and C(X,Y) in them, and need not expand these terms, unless specifically told that you need to expand them. In all cases show your work, not just the final result. If you do not show your work, you may not receive any credit even if you have the correct answer.

You will have 75 minutes to complete the exam.

1. (8 points) How many different ice cream cones can you make if there are two types of cones (waffle and sugar) and three flavors (chocolate, vanilla, and strawberry) and you can have either a 1 scoop or 2 scoop cone? *Show your work but come up with a final, numerical, answer.*

Number 1 scoop cones = 2 x 3 = 6
Number 2 scoop cones = 2 x 3 x 3 = 18
Total number cones – 6 + 18 = 24

2. (16 points) You need to create a committee of 10 people from a pool of 30 possible committee members, of which 20 are Democrats and 10 are Republicans. Unless otherwise specified, assume that all committee members serve equally. Determine the total number of ways to form each committee, based on the conditions specified below.

   For this question you may keep C(X,Y) or P(X,Y) in your answer without expanding them. Alternatively, you may generate your answer from "first principles." Either way, a numerical answer is not required (nor will you get any credit if you supply one without showing your work).

   a) There are no additional conditions—you just need to pick the 10 people.

      C(30,10) = 30!/(10! x 20!)

   b) There is a committee chair and a committee secretary but the remaining 8 members all serve equally.

C(30,1) x C(29,1) x C(28,8) *or*
P(30,1) x P(29,1) x C(28,8) *or*
P(30,2) x C(28,8) *or*
30 x 29 x 28!/(20! x 8!)

c) All 10 members serve equally but there must be an equal number of Democrats and Republicans.

C(20,5) x C(10,5)

d) All 10 members must be from a single party.

C(20,10) + C(10,10)

3. (12 points) You are given a standard deck of cards (i.e., with 52 cards) and two standard six-sided dice (i.e., with values 1 – 6). You are told to randomly select one card and roll both dice. You win $100 if you select an Ace or both dice rolls are 6's. What is the probability that you win the $100? You are not expected to simplify your answer and provide a numerical result.

P(win $100) = P(Ace) + P(double 6's) – P(Ace & double 6's)
P(Ace) = 4/52
P(double 6's) = (1/6)(1/6) = (1/36)
P(Ace & double 6's) = 4/52 x 1/36
P(win $100) = 4/52 + 1/36 – (4/52)(1/36)

If you forget about the events not being disjoint and answer 4/52 + 1/36, then you receive only 5 points.

4. (10 points) How many distinct ways can you rearrange the letters in the following words?
   a) The word "GUITAR"?

Since there are no repeats and order does matter, we get 6!

b) The word "VIOLIN"?
There are 6 total characters, and only the "I" repeats (twice).
Short way: 6!/2!

Other way: C(6,2) x C(4,1) x C((3,1) x C(2,1) x C(1,1)

5. (8 points) You flip a coin 5 times and record the result each time. What it the probability that you get at least 1 Head? You need not simplify your result.

P(at least 1 Head) = 1 – P(0 heads) = 1 – P(5 Tails)

P(5 Tails) = $(1/2)^5$ = 1/32

P(at least 1 head) = 1 – 1/32 = 31/32.

6. (8 points) What is the probability that you draw 5 cards in poker out of a standard deck of cards and get a flush? A flush means that all of the cards belong to the same suit (i.e., they are all either clubs, diamonds, hearts or spades)? As a brief review, a 52 card deck has 13 denominations (Ace, 2, 3, …, 10, J, Q, K) and 4 suits of each. You need not simplify your answer.

P(flush) = |flush|/|S|

|S| = C(52,5)
|flush| = 4 x |all clubs|
|all clubs| = C(13,5)

$$\textbf{So, } P(\text{flush}) = \frac{4 \times C(13,5)}{C(52,5)}$$

7. (15 points) A typical telephone number has 10 digits (e.g., 718-817-4495), where the first three are known as the area code and the next three as the exchange. Answer each of the following questions and make sure you show your work (you need not simplify the answer).

   a. Assuming no restrictions, how many possible (3 digit) area codes are there?

   10 x 10 x 10 = 1,000

   b. Assuming that the middle digit of the area code must be a 0 or a 1 (which I believe was required until recently), how many possible (3 digits) area codes are there?

   10 x 2 x 10 = 200

   c. Assuming no restrictions whatsoever, how many possible values are there for the full 10-digit phone number?

   10^{10} = 10,000,000,000

   d. If the only restriction is that no digit may be used more than once, how many possible 10-digit phone numbers are there?

   10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = 10!

   e. If all of the 10 digits are chosen randomly without any restrictions, what is the probability that all of the digits in the phone number will be different?

   P(E) = |E| / |S|, |E| = 10! (answer from part d) and $|S| = 10^{10}$ (answer from part c), so

   $P(E) = 10!/10^{10}$