Homework 0
(110 points total)
due September 6

Question 1: (16 points) Examine the following “set-builder” descriptions of the following sets, and provide a list of set members.

**Example “builder” description:** \{ y \mid y=3x \text{ and } x \in \mathbb{N} \}

**Example answer:** \{3, 6, 9, 12, \ldots\}

a. \{(y, 2y) \mid y \in \mathbb{N}\}

b. \{(\frac{x}{2}) \mid x \in \mathbb{Z}\}

c. \{w \mid w \text{ is a string of As and Zs, and } w \text{ equals the reverse of } w\}

d. \{4m \mid m \in \mathbb{N} \text{ and } m<5\}

Question 2: (20 points) Provide a “set-builder” description (see Question 1) for each of the sets with elements listed below.

a. \{10, 100, 1000, 10000, \ldots\}

b. \{\}\n
c. \{3, 4, 5, 6, 7, 8, \ldots\}

d. \{1, 5, 9, 13, 17, \ldots\}

e. \{2,3,4,5,6\}

Question 3: (28 points) Let A=\{ab, aabb, aaabbb, aaaaabbbb\}, B=\{ab, ababab, abababab\}, and C=\{ab, aabb\}

a. What is \(A \cup C\)?

b. True or False: \(B \subseteq A\)

c. True or False: \(C \subseteq A\)

d. What is \(A \cap B\)?

e. What is the power set of C?

For 3f and 3g, explain your answer without explicitly listing and counting all the member of the power and Cartesian-product sets.

f. How many elements are in the power set of A?

g. How many elements are in the “Cartesian product” (also known as the “set product”) \(C \times B\)?
Question 4: (14 points) Let \( X \) be the set \{2, 4, 6, 8, 10\} and \( Y \) be the set \{1, 2, 3, 4, 5\}. The unary function \( f: X \rightarrow Y \) and the binary function \( g: X \times Y \rightarrow X \) are described in the following tables.

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a. What is the value of \( f(8) \)?

b. What are the range and domain of \( f \)?

c. What is the value of \( g(8,4) \)?

d. What is the value of \( f(g(6,5)) \)?

Question 5: (12 points) Consider the undirected graph \( G=(V,E) \) where \( V \), the set of nodes, is \{1, 2, 3, 4, 5\} and \( E \), the set of edges, is \{\{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,5\}, \{3,4\}\}.

a. Draw the graph \( G \).

b. What are the degrees of each node?

c. Write a set of edges forming a path from node 1 to node 5 in the graph.

Question 6: (10 points) Write a formal description of the following graph.

![Graph Diagram]

Question 7: (10 points) Show that every graph with two or more nodes contains at least two nodes that have equal degrees. (Note, we do not allow an edge from a node to itself.)

Hint: Think of the pigeonhole principle (taught in CISC 1100/1400)!