Homework 1
Due September 20

1. Consider the state diagrams for two DFAs, M1 and M2

![State diagrams for M1 and M2](image)

a. Give the formal descriptions of the two machines above – specifically, specify the elements of the 5-tuple \((Q, \Sigma, \delta, q_0, F)\). For each machine \(Q = \?\), \(\Sigma = \?\), \(\delta = \?\), \(q_0 = \?\), \(F = \?)

b. What sequence of states does the machine go through on input yyyx

c. Are there an infinite number of strings accepted by the machine?

d. Does the machine accept the string xxyyxx?

2. The formal description of a DFA M3 is \((\{q_1, q_2, q_3, q_4, q_5\}, \{0,1\}, \delta, q_2, \{q_1, q_4\})\) where \(\delta\) is provided by the following table

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>q_1</td>
<td>q_3</td>
<td>q_4</td>
</tr>
<tr>
<td>q_2</td>
<td>q_1</td>
<td>q_5</td>
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<tr>
<td>q_3</td>
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<td>q_4</td>
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<td>q_5</td>
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<tr>
<td>q_5</td>
<td>q_1</td>
<td>q_4</td>
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</tbody>
</table>

Give the state diagram for this machine.
3. Not all context free grammars define regular languages. However, the following two do define regular languages. For each grammar below, (a) describe the corresponding language (optimally in 10 words or fewer) and (b) draw the corresponding finite state machine.

\[ G_1 \rightarrow A \mid B \]
\[ A \rightarrow xA \mid z \]
\[ B \rightarrow y \mid w \]

\[ G_2 \rightarrow CD \]
\[ C \rightarrow nnC \mid nn \]
\[ D \rightarrow pp \]

3. Give state diagrams of DFAs recognizing the following languages. In all parts, \( \Sigma = \{0,1\} \).

   a. The empty set
   b. \( \{w \mid \text{the digits interpreted in binary are evaluated to 8 or higher}\} \)
   c. \( \{w \mid w \text{ has an even number of digits}\} \)
   d. \( \{w \mid \text{the digits together sum to a number less than 3}\} \)

4. 
   a. Show that if \( M \) is a DFA that recognizes language \( B \), swapping the accept and nonaccept states in \( M \) yields a new DFA recognizing the complement of \( B \). Conclude that the class of regular languages is closed under complement.
   b. Show by giving an example that if \( M \) is an NFA that recognizes language \( C \), swapping the accept and nonaccept states in \( M \) doesn’t necessarily yield a new NFA that recognizes the complement of \( C \). Is the class of languages recognized by NFAs closed under complement? Explain your answer.

5. Consider the alphabet \( \Sigma = \{0,1,2\} \). Given this alphabet, let \( A_k = \{w \mid w \text{ is any sequence of numbers such that the number at the } k^{\text{th}} \text{ position is larger than the number at all previous positions}\} \). For example, \( A_3 \) includes 0010, 0011, 0012, 1120, 1022; \( A_4 \) includes 01021, 11120, 00012, 110221.

Show that for each \( k > 1 \), the language \( A_k \) is regular.
6. Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts, \( \Sigma = \{c, d\} \)

a. Accepts \( \{w \mid w \text{ has exactly 1 } d \text{'s and an even number of } c \text{'s}\} \) using four states.
   (Example accept strings: \( ccd, cdccc, d \})

b. Accepts \( \{w \mid w \text{ ends in } cdc\} \) using four states.
   (Example accept strings: \( cccdc, ddcdc \})

7. Use Theorem 1.45 in the text (also given in class) to provide an NFA state diagram for a machine that recognize the union of the languages \( L_1 \) and \( L_2 \) below. Note \( \Sigma = \{0, 1\} \).
   \( L_1 = \{w \mid \text{every even position in } w \text{ has 1}\} \)
   \( L_2 = \{w \mid w \text{ has 3 or more characters, and the second symbol is 1}\} \)

8. Use Theorem 1.47 in the text (also given in class) to provide an NFA state diagram for a machine that recognizes the concatenation of the languages in 3c and 3d.

9. Convert the following NFA to a DFA. You may wish to use the lecture/class method used to prove NFA-DFA equivalence.