1. For each question, your explanation can be a single sentence.

Consider the following DFA $M$:

![DFA Diagram]

a. Is $\langle M, 0100 \rangle \in A_{DFA}$? Explain.

b. Is $\langle M, 1110 \rangle \in E_{DFA}$? Explain.

c. Is $\langle M, M' \rangle \in EQ_{DFA}$? Explain.

d. Is $\langle M, 1010 \rangle \in A_{DFA}$? Explain.

Consider the following three CFG $G$:

- $S \to xA\tilde{z}$
- $A \to y \mid nA$

- $\langle G, xyz \rangle \in A_{DFA}$? Explain.

- $\langle G \rangle \in B_{CFG}$? Explain.

2. Suppose we know language $L$ is Turing recognizable. Is language $L$ co-Turing recognizable? Explain your answer.

3. A “useless state” in a pushdown automaton is never entered on any input string. Consider the problem of determining whether a pushdown automaton has any useless states. Formulate this problem as a language and show that it is decidable.
4. Let \( F = \{ <M> \mid M \text{ is a DFA that accept any string that begins with two } 0 \text{s} \} \). Show that \( F \) is decidable.

5. Consider the problem of determining whether two Turing machines are equivalent. Express this problem as a language and show that it is not decidable.

6. Let \( X_{\text{DFA}} = \{ <M,w> \mid M \text{ is a DFA and } L(M) \text{ contains at } w \text{ as a substring} \} \). If \( w=\text{abba}, L(M) \) could contain a string such as \( \text{cabbac} \) or \( \text{bbabba} \). Show that \( X_{\text{DFA}} \) is decidable.

7. For each of the problems pairs below, indicate which problem reduces to which other problem.
   a. 1: Deleting user account \( X \) from server 2: Finding user account \( X \) in server
   b. 1: Making a phone call 2: Ordering a pizza
   c. 1: Juggling a ball 2: Catching a ball

8. Consider a Turing Machine working on a tape \( T \) containing single digits \([0, 1, 2, \ldots, 8, 9]\). We will consider an algorithm I will call “crazy sort” that simply crosses out any digits not following the desired largest-to-smallest order along the tape. If your input tape started as:
   
   \[
   0 \ 2 \ 1 \ 4 \ 2 \ 4 \ 6 \ 8 \ 3 \ 1 \ 9
   \]

   the algorithm would finish with tape contents
   
   \[
   0 \ 2 \ x \ 4 \ x \ 4 \ 6 \ 8 \ x \ x \ 9
   \]

   The algorithm proceeds as follows
   
   1. Mark left-most digit
   2. Repeat the following loop until no more digits left to consider on tape
      a. Scan across tape, crossing off all digits less than the marked off digit
      b. Move digit marker one digit to the right of currently marked digit on tape

   a. What is the Big-O complexity of this method?

   b. Propose a method that will implement “crazy sort” with smaller Big-O complexity.
9.
Consider the following $o(n)$ and $O(n)$ complexities:

- $o(n)$
- $o(n \log n)$
- $o(n^2)$
- $o(2^n)$
- $O(1)$
- $O(\log n)$
- $O(n^4)$
- $O(2^n)$

For each of the following $f(n)$ functions, list the smallest correct $O(n)$ and $o(n)$ complexities from the above list (if any).

a. $f(n) = 20n + 3n^3$

b. $f(n) = \log n + n$

c. $f(n) = 300n^3 + 2^n$

d. $f(n) = 50n + 2 \log n$

e. $f(n) = 50n^3 \log n + 20n^2$

f. $f(n) = 5/n^2$