1. Consider the state diagram for the following DFAs. For each, answer the following questions:
(1) What state is reached by the input: \( w=00110 \) ?
(2) What is the transition function?
(3) What is the language recognized?

**M1:**

![State Diagram M1](image1)

- (1) \( q_2 \) (0) \( \rightarrow \) \( q_0 \) 0 \( \rightarrow \) \( q_0 \) 1 \( \rightarrow \) \( q_1 \) 1 \( \rightarrow \) \( q_2 \) 0 \( \rightarrow \) \( q_2 \)
- (2) |
<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_0 )</td>
<td>( q_0 )</td>
</tr>
<tr>
<td>( q_1 )</td>
<td>( q_1 )</td>
</tr>
<tr>
<td>( q_2 )</td>
<td>( q_2 )</td>
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</tbody>
</table>
- (3) \{ w \mid \text{the number of 1's entered is a multiple of 3} \}
  \{0^*(10^*10^*10^*)^*\}

**M2:**

![State Diagram M2](image2)

**M3:**

![State Diagram M3](image3)
2. Define a machine to recognize the following languages in the alphabet 
\( \Sigma = \{1,2,3\} \) 
(5 points) 

- **L4** = \{ \( w \) | the product of input symbols is even \}  
  E.g., 111 -> 1x1x1 = 1 is odd-reject,  
  233 -> 2x3x3 = 18 is even-accept 

- **L5** = \{ \( w \) | numbers entered in non-decreasing order \}  
  Examples: 112223, 122333 

- **L6** = \{ \( w \) | first two symbols are identical \}  
  Examples: 001213, 333212, 3310013 

3. Prove the following languages are regular, using the alphabet \( \Sigma = \{a, b, c\} \): 

- **L7** = \{ \( w \) | \( w \) contains an odd number of b’s \}  
  Define a DFA to detect the language and/or show a regular expression captures the language.
\[ (aUc)^*b(aUc)^*((aUc)^*b(aUc)^*b(aUc)^*) \]

\[ L_8 = \{ w \mid w \text{ contains the sequence } bcb \} \quad (\text{Examples: } aabbbcb or ccbbcba) \]

\[ L_9 = \{ w \mid w \text{ does not have three a’s in a row} \} \]
4. Consider the following NFAs. For each, answer:

(1) what state(s) will be reached by the input: 0011
(2) provide a regular expression to describe the recognized language
(3) For N11 and N12, convert NFA to DFA

N10:

(1) No state – it will be rejected!
(2) $1(0^* U 0^* (0^*01)^*)$
5. For each regular expression using $\Sigma = \{a, b\}$:
(1) Provide three example words.
(2) Convert these regular expressions to a DFA or NFA

$L_{13} = \{ab^*(ba)^*\}$

$L_{14} = \{(a \cup b)ba^*\}$

$L_{15} = \{(bb)^* \cup (aa)^*\}$
(1) Examples: $bb$, $aa$, $bbbbbb$, $aaaaaa$, $\varepsilon$
(2)

6. What is the minimum pumping length for each of these languages, showing these languages are regular? We use the alphabet $\Sigma = \{0, 1, 2\}$

$L_{16} = \{00(0 \cup 1)^*12\}$

$L_{17} = \{0(22)^*10\}$
$p = 5$, minimum pumpable string is $02210$

$L_{18} = \{111(202)^*210\}$
If pumping length is \( p = 5 \), how would you break up string \( w \) into \( x, y, \) and \( z \) for languages \( L \) below?

\[ L_{19} = \{ 20(11)^*001 \}, \quad w = 201111001 \]

\[ L_{20} = \{ (121)^*001 \} \quad w = 121001 \]

7. Consider the language \( L_{21} = \{ 01(101)^*11 \} \), what is the error in each of the following “pumping lemma” arguments?

**Argument 1:** Let us take \( w = 0111 \), \( w \in L_{21} \). We cannot divide \( w = xyz \) such that \( y^iz \in L_{21}, i \geq 0 \). For example, if \( x = 0, y = 11, \) and \( z = 1 \), \( xy^2z = 011111 \notin L_{21} \). Therefore, \( L_{21} \) is not regular.

The pumping length is \( p = 7 \). Using any strings in \( L_{21} \) with length less than pumping length is not necessarily pumpable, and the inability to pump a too-short string does not prove anything. You can only test pumping on strings with at least as many characters as the pumping length.

**Argument 2:** Let us take \( w = 0110110111 \), \( w \in L_{21} \). If we divide \( w = xyz \) as follows: \( x = 0110110, y = 11, z = 1 \), we cannot repeat \( y \) such that \( xy^iz \in L_{21}, i \geq 0 \). For example, if \( xy^2z = 011011011111 \notin L_{21} \). Therefore, \( L_{21} \) is not regular.
8. Prove these languages are not regular.

$L_{24} = \{0^n1^{2n}0^{3n} \mid n > 0 \}$

Proof by contradiction with pumping lemma:
Assume $L_{24}$ is pumpable. Now consider $w = 0^p2^p0^{3p}$, which is element of $L_{24}$ with $|w| > p$. Thus, $w$ must be pumpable.

Let $w = xyz$ where $x = 0^j$, $y = 0^k$, $z = 0^{(p-(j+k))}2^p0^{3p}$, and $j + k \leq p$.

Try pumping $w$: $xy^2z$ begins with $j + 2k + p - (j + k) \geq 0$'s, $j + 2k + p - (j + k) = p + k$ 0's.

$xy^2z$ begins with $p + k$ 0's followed by $2p$ 1's.

$2p \neq p + k$, so $xy^2z \not\in L_{24}$, which means $L_{24}$ is not regular!

$L_{25} = \{1^n3^n \mid n > 0 \}$

9. For each of the following grammars, list three strings produced by the grammar

$G_{26}$:

$S \rightarrow AB \mid BA$
$A \rightarrow xAy \mid \varepsilon$
$B \rightarrow BzB \mid y$

Examples:
$AB \rightarrow \varepsilon y \rightarrow y$
$BA \rightarrow yxAy \rightarrow yxxAyy \rightarrow yxyyy \rightarrow yxyy$
$AB \rightarrow \varepsilon BzB \rightarrow \varepsilon BzBzy \rightarrow \varepsilon BzBzyy \rightarrow \varepsilon yzyzyy$

$G_{27}$:

$S \rightarrow A \mid AA$
$A \rightarrow 00 \mid 11$

$G_{28}$:

$A \rightarrow 11A00 \mid \varepsilon$
10. Provide the languages described by two of the grammars:

G27 (from above)

G28 (from above)

10. Provide a grammar to produce the following languages

L32 = \{0^n(11)^n | n \geq 0\}

L33 = \{01^*00^*\}

L34 = \{w | w=\text{w}^{\text{Reverse}}\} 
Examples: 00100, 10101, 1111
S -> 0S0 | 1S1 | 0 | 1 | \epsilon

11. Convert the following grammars to Chomsky Normal Form

G29:
S -> xAy | BA
A -> z | AzA
B -> yB | ε

Remove the ε
S -> xAy | BA | A
A -> z | AzA
B -> yB | y

Remove the S->A unit rule
S -> xAy | BA | z | AzA
A -> z | AzA
B -> yB | y

Replace mixed terminal—variable rules with variables-only rules
S -> UxAUy | BA | z | AUzA
Ux -> x
Uy -> y
Uz -> z
A -> z | AUzA
B -> UyB | y

Replace 3\* variable rules with 2-variables rules
S -> UxC | BA | z | AD
C -> AUy
D -> UzA
Ux -> x
Uy -> y
Uz -> z
A -> z | AD
B -> U,yB | y

G30:
S -> BAB | ABA
A -> y | z
B -> x | AA | ε

G31:
S -> ByBy
B -> xBx | ε