1. Consider the state diagram for the following DFAs. For each, answer the following questions:
(1) What state is reached by the input: w=00110 ?
(2) What is the transition function?
(3) What is the language recognized?

**M1:**

![State Diagram for M1](image)

(1) \( q_2 \) (0) -> \( q_0 \) 0 -> \( q_0 \) 1 -> \( q_1 \) (1) -> \( q_2 \) (0) -> \( q_2 \)

(2) |
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<td>1</td>
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<tr>
<td>( q_2 )</td>
<td>0</td>
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(3) \( \{w | \text{the number of 1's entered is a multiple of 3}\} \)
   \( \{0^*(10^*10^*)^*\} \)

**M2:**

![State Diagram for M2](image)

(1) \( q_2 \) (0) -> \( q_0 \) 0 -> \( q_0 \) 1 -> \( q_1 \) (1) -> \( q_2 \) (0) -> \( q_2 \)

**M3:**
2. Define a machine to recognize the following languages in the alphabet 
\( \Sigma = \{1, 2, 3\} \)
(5 points)

- \( L_4 = \{ w \mid \text{the product of input symbols is even} \} \)  
  E.g., 111 -> 1x1x1 = 1 is odd-reject,  
  233 -> 2x3x3 = 18 is even-accept

- \( L_5 = \{ w \mid \text{numbers entered in non-decreasing order} \} \)  
  Examples: 112223, 122333

- \( L_6 = \{ w \mid \text{first two symbols are identical} \} \)  
  Examples: 001213, 333212, 3310013

3. Prove the following languages are regular, using the alphabet \( \Sigma = \{a, b, c\} \):

- \( L_7 = \{ w \mid w \text{ contains an odd number of } b's \} \)
  Define a DFA to detect the language and/or show a regular expression captures the language.
L8={w | w contains the sequence bcb}  (Examples: aabbbcb or cccbcba)

L9={w | w does not have three a’s in a row}
4. Consider the following NFAs. For each, answer:
   (1) what state(s) will be reached by the input: 0011
   (2) provide a regular expression to describe the recognized language
   (3) For N11 and N12, convert NFA to DFA

N10:

(1) No state – it will be rejected!
(2) 1(0* U 0* (0*01)*)

N11:

N12:
5. For each regular expression using $\Sigma = \{a, b\}$:
   (1) Provide three example words.
   (2) Convert these regular expressions to a DFA or NFA

$L13 = \{ab^*(ba)^*\}$

$L14 = \{(a \cup b)ba^*\}$

$L15 = \{(bb)^* \cup (aa)^*\}$
(1) Examples: $bb$, $aa$, $bbbbbb$, $aaaaaa$, $\varepsilon$
(2)

![DFA/NFA Diagram]

6. What is the minimum pumping length for each of these languages, showing these languages are regular? We use the alphabet $\Sigma = \{0, 1, 2\}$

$L16 = \{00(0 \cup 1)^*12\}$

$L17 = \{0(22)^*10\}$
$p=5$, minimum pumpable string is $02210$

$L18 = \{111(202)^*210\}$
If pumping length is $p=5$, how would you break up string $w$ into $x$, $y$, and $z$ for languages $L$ below?

$L_{19} = \{20(11)^*001\}$, \hspace{1cm} w=20111001

$L_{20} = \{(121)^*001\}$ \hspace{1cm} w=121001

7. Consider the language $L_{21} = \{01(101)^*11\}$, what is the error in each of the following “pumping lemma” arguments?

**Argument 1:** Let us take $w=0111$, $w \in L_{21}$. We cannot divide $w=xyz$ such that $y^iz \in L_{21}$, $i \geq 0$. For example, if $x=0$, $y=11$, and $z=1$, $xy^2z = 011111 \notin L_{21}$. Therefore, $L_{21}$ is not regular.

**The pumping length is $p=7$. Using any strings in $L_{21}$ with length less than pumping length is not necessarily pumpable, and the inability to pump a too-short string does not prove anything. You can only test pumping on strings with at least as many characters as the pumping length.**

**Argument 2:** Let us take $w=0110110111$, $w \in L_{21}$. If we divide $w=xyz$ as follows: $x=0110110$, $y=11$, $z=1$, we cannot repeat $y$ such that $xy^iz \in L_{21}$, $i \geq 0$. For example, if $xy^2z = 011011011111 \notin L_{21}$. Therefore, $L_{21}$ is not regular.
8. Prove these languages are not regular.

$L_{24} = \{0^n1^{2n}0^{3n} \mid n > 0 \}$

Proof by contradiction with pumping lemma:
Assume $L_{24}$ is pumpable. Now consider $w = 0^p2^p0^{3p}$, which is element of $L_{24}$ with $|w| > p$. Thus, $w$ must be pumpable.

$w = xyz \quad x = 0^j \quad y = 0^k \quad z = 0^{(p-j-k)}2^p0^{3p} \quad j + k \leq p$

Try pumping $w$: $xy^2z \rightarrow 0^j0^{2k}0^{(p-j-k)}2^p0^{3p}$

$xy^2z$ begins with $j + 2k + p - (j + k)$ 0’s ... $j + 2k + p - (j + k) = p + k$ 0’s
$xy^2z$ begins with $p + k$ 0’s followed by $2p$ 1’s.

$2p \neq p + k$, so $xy^2z \notin L_{24}$, which means $L_{24}$ is not regular!

$L_{25} = \{1^n3^n \mid n > 0 \}$

9. For each of the following grammars, list three strings produced by the grammar

G26:
$S \rightarrow AB \mid BA$
$A \rightarrow xAy \mid \epsilon$
$B \rightarrow BzB \mid y$

Examples: $AB \rightarrow \epsilon y \rightarrow y$
$BA \rightarrow yxAy \rightarrow yxxAy \rightarrow yxx\epsilon yy \rightarrow yxxyy$
$AB \rightarrow \epsilon BzB \rightarrow \epsilon BzBzy \rightarrow \epsilon BzBzzy \rightarrow \epsilon\text{yzyzyzy}$

G27:
$S \rightarrow A \mid AA$
$A \rightarrow 00 \mid 11$

G28:
$A \rightarrow 11A00 \mid \epsilon$
10. Provide the languages described by two of the grammars:

G27 (from above)

G28 (from above)

10. Provide a grammar to produce the following languages

\[ L_{32} = \{0^n(11)^n \mid n \geq 0\} \]

\[ L_{33} = \{01^*00^*\} \]

\[ L_{34} = \{w \mid w = w^{reverse}\} \quad \text{Examples: 00100, 10101, 1111} \]

\[ S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid \epsilon \]

11. Convert the following grammars to Chomsky Normal Form

G29:

\[ S \rightarrow xAy \mid BA \]

\[ A \rightarrow z \mid AzA \]
B -> yB | ε

Remove the ε
S -> xAy | BA | A
A -> z | AzA
B -> yB | y

Remove the S->A unit rule
S -> xAy | BA | z | AzA
A -> z | AzA
B -> yB | y

Replace mixed terminal—variable rules with variables-only rules
S -> UxAY | BA | z | AUzA
Ux -> x
UY -> y
Uz -> z
A -> z | AUzA
B -> UyB | y

Replace 3* variable rules with 2-variables rules
S -> UxC | BA | z | AD
C -> AUY
D -> UzA
Ux -> x
UY -> y
Uz -> z
A -> z | AD
B -> UyB | y

G30:
S -> BAB | ABA
A -> y | z
B -> x | AA | ε

G31:
S-> ByBy
B -> xBx | ε