f1. Consider the state diagram for the following DFAs. For each, answer the following questions:
(1) What state is reached by the input: w=00110 ?
(2) What is the transition function?
(3) What is the language recognized?

M1:

M2:

M3:

(1) \(q_3\)  
\(q_0\) (0) -> \(q_2\) (0) -> \(q_3\) (1) -> \(q_3\) (1) -> \(q_3\) (0) -> \(q_3\)

(2)

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(3) \(L_3 = \{w \mid \text{contains exactly one 0, followed by no other symbols}\} \quad 1^*0\)
2. Define a machine to recognize the following languages in the alphabet \( \Sigma = \{1,2,3\} \)
(5 points)

**L4**\(=\{w \mid \text{the product of input symbols is even}\} \)
E.g., \(111 \rightarrow 1 \times 1 \times 1 = 1 \) is odd-reject,
\(233 \rightarrow 2 \times 3 \times 3 = 18 \) is even-accept

Presuming initial running product is 1:

\[(1U3)\ast 2(1U2U3)\ast\]

**L5**\(=\{w \mid \text{numbers entered in non-decreasing order}\} \)
Examples: 112223, 122333

**L6**\(=\{w \mid \text{first two symbols are identical}\} \)
Examples: 001213, 333212, 3310013

3. Prove the following languages are regular, using the alphabet \( \Sigma = \{a, b, c\} \):

**L7**\(=\{w \mid \text{w contains an odd number of b's}\} \)

**L8**\(=\{w \mid \text{w contains the sequence bcb}\} \) (Examples: aabbbcbba or ccbbcba)
Prove this by showing an FSA that recognizes L8 or showing regular expression for L8

NFA:

RegExp: \((aUbUc)\ast bcb(aUbUc)\ast\) or \(\Sigma\ast bcb\Sigma\ast\)

**L9**\(=\{w \mid \text{w does not have three a's in a row}\} \)
4. Consider the following NFAs. For each, answer:
(1) what state(s) will be reached by the input: 0011
(2) provide a regular expression to describe the recognized language
(3) For N11 and N12, convert NFA to DFA

**N10:**

**N11:**

**N12:**

(1) Enter \{\} state (exit all listed states for NFA)
(2) $1^*0(01)^*$
(3)
5. For each regular expression using $\Sigma = \{a, b\}$:
   (1) Provide three example words.
   (2) Convert these regular expressions to a DFA or NFA

$L_{13} = \{aba^*(ba)^*\}$

$L_{14} = \{(a \cup b)ba^*\}$
   (1) Examples: $ab, bb, aba, bba, abaa, bbab$

$L_{15} = \{(bb)^* \cup (aa)^*\}$

6. What is the minimum pumping length for each of these languages, showing these languages are regular? We use the alphabet $\Sigma = \{0, 1, 2\}$

$L_{16} = \{00(0 \cup 1)^*12\}$

$L_{17} = \{0(22)^*10\}$

$L_{18} = \{111(202)^*210\}$
   $p=9$ smallest pumpable word: 111202210
If pumping length is \( p = 5 \), how would you break up string \( w \) into \( x \), \( y \), and \( z \) for languages \( L \) below?

\[
L_{19} = \{ 20(11)^*001 \}, \quad w = 20111001 \\
x = 20 \quad y = 11 \quad z = 11001
\]

\[
L_{20} = \{ (121)^*001 \} \quad w = 121001
\]

7. Consider the language \( L_{21} = \{01(101)^*11\} \), what is the error in each of the following “pumping lemma” arguments?

**Argument 1:** Let us take \( w = 0111, \ w \in L_{21} \). We cannot divide \( w = xyz \) such that \( y^iz \in L_{21}, \ i \geq 0 \). For example, if \( x = 0, \ y = 11, \) and \( z = 1, \ xy^2z = 011111 \notin L_{21} \). Therefore, \( L_{21} \) is not regular.

**Argument 2:** Let us take \( w = 0110110111, \ w \in L_{21} \). If we divide \( w = xyz \) as follows: \( x = 0110110, \ y = 11, \ z = 1 \), we cannot repeat \( y \) such that \( xy^iz \in L_{21}, \ i \geq 0 \). For example, if \( xy^2z = 011011011111 \notin L_{21} \). Therefore, \( L_{21} \) is not regular.

8. Prove these languages are not regular.

\[
L_{24} = \{ 0^n1^{2n}0^n \mid n > 0 \} \]
L25=$\{1^n^3 \mid n>0\}$
Proof by contradiction:
Assume L25 is regular. Consider $w=1^{p^3}$ $w \in L25$ $\mid w \mid > p$
$x=1^j$ $y=1^k$ $z=1^{p^3-(j+k)}$
Try pumping: $xy^2z \rightarrow 1^j 1^k 1^{p^3-(j+k)} \rightarrow$
Total number of 1’s: $j+k+p^3-(j+k) = p^3+k$
Next word after $1^{p^3}$ will be $1^{(p+1)^3}$
Size of next-biggest word: $(p+1)^3 = (p^2+2p+1)(p+1) = p^3+3p^2+3p+1$
$k \leq 0$, so $p^3+k < p^3+3p^2+3p+1$ $p^3+k<(p+1)^3$
Therefore, pumped $w$ is not in L25.

9. For each of the following grammars, list three strings produced by the grammar

G26:
$S \rightarrow AB \mid BA$
$A \rightarrow xAy \mid \epsilon$
$B \rightarrow BzB \mid y$

G27:
$S \rightarrow A \mid AA$
$A \rightarrow 00 \mid 11$
Examples: $A \rightarrow 00$, $A \rightarrow 11$, $AA \rightarrow 0011$

G28:
$A \rightarrow 11A00 \mid \epsilon$

10. Provide the languages described by two of the grammars:
G27 (from above)
$\{00U11\}(00U11)^*$
$00 \ U \ 11 \ U \ (00U11)(00U11)$
G28 (from above)

10. Provide a grammar to produce the following languages

L32 = {0^n(11)^n | n\geq 0}
S -> OS11 | \epsilon

L33 = {01^*00^*}

L34 = {w | w=w^{Reverse}}   Examples: 00100, 10101, 1111

11. Convert the following grammars to Chomsky Normal Form

G29:
S -> xAy | BA
A -> z | AzA
B -> yB | \epsilon

G30:
S -> BAB | ABA
A -> y | z
B -> x | AA | \epsilon
G31:
S -> ByBy
B -> xBx | ε

Answer corrected March 5, 10am
Replace ε:
S -> ByBy | Byy | yBy | yy
B -> xBx | xx

Replace terminal--variable rules with all-variable rules:
S -> BUyBUy | BUyUy | UyBUy | UyUy
Uy -> y
B -> UxBu | UxUx
Ux -> x

Replace 3+ variable rules with 2-variable rules
S -> BC | BE | UyD | UyUy
C->UyD
D->BUy
E->UyUy
Uy -> y
B -> UxF | UxUx
F -> BUx
Ux -> x

Old answer with errors
Replace ε:
S -> ByBy | ByB | BB | BB
B -> xBx | xx

Replace terminal--variable rules with all-variable rules:
S -> BUyBUy | BUyUy | UyBUy | UyB | BB
Uy -> y
\[ B \rightarrow U_x BU_x \mid U_x U_x \]
\[ U_x \rightarrow x \]

Replace 3-variable rules with 2-variable rules:

\[ S \rightarrow BC \mid BD \mid BE \mid BB \]
\[ C \rightarrow U_x D \]
\[ D \rightarrow BU_x \]
\[ E \rightarrow U_x B \]
\[ U_x \rightarrow y \]
\[ B \rightarrow U_x E \mid U_x U_x \]
\[ F \rightarrow BU_x \]
\[ U_x \rightarrow x \]