1. Consider the state diagram for the following DFAs. For each, answer the following questions:
(1) What state is reached by the input: $w=00110$?
(2) What is the transition function?
(3) What is the language recognized?

**M1:**

(1) $q_3$  ($q_0$ (0) -> $q_3$ (0) -> $q_4$ (1) -> $q_3$ (1) -> $q_4$ (0) -> $q_3$)
(2)

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(3) $\{w \mid w$ is $0$ followed by odd number of digits or $w$ is $1$ followed by even number of digits$\} = \{1((0 \cup 1)(0 \cup 1))^* \cup 0((0 \cup 1)(0 \cup 1))^* \cup 0(0 \cup 1)((0 \cup 1)(0 \cup 1))^* \}$

**M2:**
2. Define a machine to recognize the following languages in the alphabet $\Sigma = \{1,2,3\}$ (5 points)

$L4=\{w \mid$ the product of input symbols is even$\}$  E.g., $111\rightarrow 1\times 1\times 1 = 1$ is odd-reject, $233 \rightarrow 2\times 3\times 3 = 18$ is even-accept

$L5=\{w \mid$ numbers entered in non-decreasing order$\}$  Examples: 112223, 122333

$L6=\{w \mid$ first two symbols are identical$\}$  Examples: 001213, 333212, 3310013

3. Prove the following languages are regular, using the alphabet $\Sigma = \{a, b, c\}$:

$L7=\{w \mid w$ contains an odd number of b's$\}$

$L8=\{w \mid w$ contains the sequence bcb$\}$ (Examples: aabbbcb or ccbbca)
L9 = \{w \mid w \text{ does not have three a's in a row}\}
Construct a DFA that recognizes three a's in a row, then flip the accept and reject states.

4. Consider the following NFAs. For each, answer:
   (1) what state(s) will be reached by the input: 0011
   (2) provide a regular expression to describe the recognized language
   (3) For N11 and N12, convert NFA to DFA

N10:

N11:
(1) $q_2 \rightarrow q_0 \rightarrow q_1 \rightarrow q_0 \rightarrow q_1 \rightarrow q_2$

(2) $(0U1)(0(0U1))^*1$ or alternatively: $\Sigma(0\Sigma)^*1$

3. For each regular expression using $\Sigma = \{a, b\}$:
(1) Provide three example words.
(2) Convert these regular expressions to a DFA or NFA

$L13 = \{ab^* (ba)^*\}$

(1) a, ababa, abbb, abbbaba

(2) CORRECTED OCTOBER 16, 11:45pm
\[ L_{14} = \{(a \cup b)ba^*\} \]

\[ L_{15} = \{(bb)^* \cup (aa)^*\} \]

6. What is the minimum pumping length for each of these languages, showing these languages are regular? We use the alphabet \( \Sigma = \{0,1,2\} \)

\[ L_{16} = \{00(0 \cup 1)^*12\} \]

\( p=5 \): 00012 or 00112 are the smallest strings you can pump

\[ L_{17} = \{0(22)^*10\} \]

\[ L_{18} = \{111(202)^*210\} \]

If pumping length is \( p=5 \), how would you break up string \( w \) into \( x, y, \) and \( z \) for languages \( L \) below?

\[ L_{19} = \{20(11)^*001\}, \quad w=20111001 \]

\[ L_{20} = \{(121)^*001\} \quad w=121001 \]

\( x=\epsilon \quad y=121 \quad z=001 \)

7. Consider the language \( L_{21} = \{01(101)^*11\} \), what is the error in each of the following “pumping lemma” arguments?
Argument 1: Let us take \( w=0111, w \in L_{21} \). We cannot divide \( w=xyz \) such that \( xy^iz \in L_{21}, i \geq 0 \). For example, if \( x=0, y=11, \) and \( z=1 \), \( xy^2z = 011111 \notin L_{21} \). Therefore, \( L_{21} \) is not regular.

Argument 2: Let us take \( w=0110110111, w \in L_{21} \). If we divide \( w=xyz \) as follows: \( x=0110110, y=11, z=1 \), we cannot repeat \( y \) such that \( xy^iz \in L_{21}, i \geq 0 \). For example, if \( xy^2z = 011011011111 \notin L_{21} \). Therefore, \( L_{21} \) is not regular.

We divided \( w \) improperly to allow pumping. There exist other divisions of \( w \) that are pumpable, such as: \( x=01, y=101, z=10111 \).

8. Prove these languages are not regular.

\( L_{24} = \{0^n1^{2n}0^{3n} \mid n>0 \} \)

\( L_{25} = \{1^n3^3 \mid n>0\} \)

9. For each of the following grammars, list three strings produced by the grammar

**G26:**
- \( S \rightarrow AB \mid BA \)
- \( A \rightarrow xAy \mid \varepsilon \)
- \( B \rightarrow BzB \mid y \)

**G27:**
S -> A | AA
A -> 00 | 11

G28:
A -> 11A00 | \( \varepsilon \)

Examples: \( \varepsilon \), 1100, 1110000, 11111000000

10. Provide the languages described by two of the grammars:

G27 (from above)

G28 (from above)

\[ L = \{(11)^n(00)^n \mid n \geq 0\} \]

10. Provide a grammar to produce the following languages

\( L_{32} = \{0^n(11)^n \mid n \geq 0\} \)

\( L_{33} = \{01^*00^*\} \)

Think of a standard DFA and convert from there

G33:
S -> 0R₁
R₁ -> 1R₁ | 0R₂
R₂ -> 0R₂ | \( \varepsilon \)

\[ L_{34} = \{w \mid w = w^{\text{Reverse}}\} \]

Examples: 00100, 10101, 1111
11. Convert the following grammars to Chomsky Normal Form

**G29:**

S → xAy | BA  
A → z | AzA  
B → yB | ε

**G30:**

S → BAB | ABA  
A → y | z  
B → x | AA | ε

Remove ε  
S → BAB | ABA | AB | BA | A | AA  
A → y | z  
B → x | AA

Remove unit S→A  
S → BAB | ABA | AB | BA | y | z | AA  
A → y | z  
B → x | AA

Convert 3+ variable rules to chain of 2-variable rules  
S→BC | AD | AB | BA | y | z | AA  
C → AB  
D → BA  
A → y | z  
B → x | AA
G31:
S -> ByBy
B -> xBx | ε