

Homework on Logic
CISC 1100
Fall 2009
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1. Let A, B, C and D by the following statements:
A : The villain is French.
B: The hero is American.
C: The heroine is British.
D: The movie is good.

Translate the following propositional forms into English sentences:

a. $B \vee \neg C$

b. $A \vee (B \wedge \neg C)$

c. $(C \wedge \neg A) \Leftrightarrow B$

2. Write each statement as a statement of formal propositional forms, that is, assign variable names to the simple propositions, and write the statement using those variables along with logic operator such as $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$. Then negate the form, and simplified it.

For example, for statement “I ate my lunch but I did not eat breakfast.”

Let's use “l” to denote “I ate my lunch”, and use “b” to denote “I ate my breakfast”. Then the whole statement can be expressed as

$$l \wedge \neg b$$

The negation of the statement, i.e., it's not true that I ate my lunch but did not eat breakfast, is awkward. So let's see if we can simplify it:

$$\neg(l \wedge \neg b) = \neg l \vee (\neg \neg b) = \neg l \vee b.$$

3. For each statement in Question 2, give a truth table for the statement, and use the truth table to explain what conditions would make the statement true and what would make it false.

4. Construct truth table for each of the following propositional forms. By inspection of the truth tables, answer which are tautologies and which are contradiction.

a. $\neg p \vee p$;

b. $\neg(p \wedge \neg q)$;

c. $(p \Rightarrow q) \wedge (q \Rightarrow p)$;

d. $\neg p \vee q$

d. $(p \Rightarrow q) \Leftrightarrow (q \Rightarrow p)$;

e. $(p \Rightarrow q) \wedge (p \Rightarrow r)$;

5. Inspect the truth tables of the forms from Question 4, write down those pairs of forms which are equivalent.

6. You met two inhabitants of Smullyan's Island, A and B. A says "I am lying if B is", and B says "A is lying if I am". Can you tell who if anyone is telling the truth?