

Sol: Homework on Logic
CISC 1100
Fall 2009
Instructor: Ellen Zhang
120 pts total

1. [15 pt, 5 pt for each sub problem] Let A, B, C and D by the following statements:
A : The villain is French.
B: The hero is American.
C: The heroine is British.
D: The movie is good.

Translate the following propositional forms into English sentences:

a. $B \vee \neg C$

Ans. The hero is American or the heroine is not British.

b. $A \vee (B \wedge \neg C)$

Ans: The villain is French, or the hero is American and the heroine is not british.

c. $(C \wedge \neg A) \Leftrightarrow B$

Ans: The heroine is British and the villain is not French, if and only if the hero is American.

2. [50 pts. 10 pt for each sub-problem: 4 pt for translating the sentence to logic form, 6 pt for negating and simplified negated form] Write each statement as a statement of formal propositional forms, that is, assign variable names to the simple propositions, and write the statement using those variables along with logic operator such as $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$. Then negate the form, and simplified it.

For example, for statement “I ate my lunch but I did not eat breakfast.”

Let’s use “l” to denote “I ate my lunch”, and use “b” to denote “I ate my breakfast”. Then the whole statement can be expressed as

$$l \wedge \neg b$$

The negation of the statement, i.e., it’s not true that I ate my lunch but did not eat breakfast, is awkward. So let’s see if we can simplify it:

$$\neg(l \wedge \neg b) = \neg l \vee (\neg \neg b) = \neg l \vee b.$$

Translate the negated form back to English, we get: I didn’t eat lunch or I ate my breakfast, which is much clearer.

- a. If you don’t attend the concert, you will get an F for the course.

Ans: Let c stands for “You attend the concert”, and f stands for “You get an F for the course”. The statement can be written as $\neg c \Rightarrow f$.

The negation of the statement is $\neg(\neg c \Rightarrow f)$. It means: it’s not the case that “If you don’t attend the concert, you will get an F for the course”. We simplify the form as follows:
 $\neg(\neg c \Rightarrow f) = \neg(\neg(\neg c) \vee f) = \neg(c \vee f) = \neg c \wedge \neg f$.

Translate the simplified form back to English, we get: You don’t attend the concert, and you do not get an F for the course.

b. We will go if you go.

Ans. Let w stands for “we go”, and y stands for “you go.”. The statement can be written as $y \Rightarrow w$.

The negation of the statement is $\neg(y \Rightarrow w)$. It means: it’s not the case that “We will go if you go”. We simplify the form as follows: $\neg(y \Rightarrow w) = \neg(\neg y \vee w) = y \wedge \neg w$.

Translate the simplified form back to English, we get: You go, but we do not go.

c. If you don’t eat your breakfast, you will be hungry.

Ans. Let b stands for “You eat your breakfast”, and h stands for “You are hungry.”. The statement can be written as $\neg b \Rightarrow h$.

The negation of the statement is $\neg(\neg b \Rightarrow h)$. It means: it’s not the case that “If you don’t eat your breakfast, you will be hungry”. We simplify the form as follows:

$$\neg(\neg b \Rightarrow h) = \neg(\neg\neg b \vee h) = b \wedge \neg h.$$

Translate the simplified form back to English, we get: You go, but we do not go.

d. It’s false that this triangle has both a 30 degree angle and a 60 degree angle.

Ans: Let p stands for “this triangle has a 30 degree angle”, and q stands for “this triangle has a 60 degree angle”. The statement can be written as $\neg(p \wedge q)$.

The negation of the statement is $\neg\neg(p \wedge q)$, which is equivalent to $(p \wedge q)$.by double negation law. Translate the simplified form back to English, we get: This triangle has both a 30 degree angle and a 60 degree angle.

3. [20 pts, 4 pts for each sub-problem] For each statement in Question 2, give a truth table for the statement, and use the truth table to explain what conditions would make the statement true and what would make it false.

a. $\neg c \Rightarrow f$.

c		$\neg c$	$\neg c \Rightarrow f$.
T	F	F	T
T	F	F	T
F	T	T	T
F	F	T	F

The whole statement is false when c is true and f is false, i.e., you do not go to the concert, and you do not get F for the course. Otherwise, it's true.

b. $y \Rightarrow w$.

y	w	$y \Rightarrow w$.
T	T	T
T	F	F
F	T	T
F	F	T

The above statement is false when y is true and w is false, i.e., when "you go", but "we do not go".

c. $\neg b \Rightarrow h$.

b	h	$\neg b$	$\neg b \Rightarrow h$.
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

The above statement is false when b is false, and h is false, i.e., when "you do not eat breakfast", and "you do not feel hungry".

d. $\neg(p \wedge q)$.

p	q	$p \wedge q$	$\neg(p \wedge q)$.
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

The statement is false when p and q are both true, i.e., this triangle has both a 30 degree angle and a 60 degree angle.

4. [30 pts, 5 pts for each sub-problem] Construct truth table for each of the following propositional forms. By inspection of the truth tables, answer which are tautologies and which are contradiction.

a. $\neg p \vee p$;

Ans; Truth table is omitted. It's a tautology.

b. $\neg(p \wedge \neg q)$;

Ans: Not a tautology, not a contradiction.

c. $(p \Rightarrow q) \wedge (q \Rightarrow p)$;

ANs: not a tautology, not a contradiction.

d. $\neg p \vee q$

Ans: a tautology, not a contradiction.

e. $(p \Rightarrow q) \Leftrightarrow (q \Rightarrow p)$;

Ans; a tautology, not a contradiction.

P	q	$(p \Rightarrow q)$	$(q \Rightarrow p)$;	$(p \Rightarrow q) \Leftrightarrow (q \Rightarrow p)$;
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

e. $(p \Rightarrow q) \wedge (p \Rightarrow r)$;

Ans: a tautology, not a contradiction.

P	q	r	$(p \Rightarrow q)$	$(p \Rightarrow r)$;	$(p \Rightarrow q) \wedge (p \Rightarrow r)$;
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

5. [5 pts] Inspect the truth tables of the forms from Question 4, write down those pairs of forms which are equivalent.

Ans: b) and d) are equivalent.

6. [10 pts. 5 pts for express A's statement and B's statement in logic form. 2 pts for drawing the truth table, 3 pts for giving the final answer.] You met two inhabitants of Smullyan's Island, A and B. A says "I am lying if B is", and B says "A is lying if I am". Can you tell who if anyone is telling the truth?

Ans: Let a stands for "A is truth-teller", b stands for "B is truth-teller". Then what A says can be written as $b \Rightarrow a$. What B says can be written as $b \Rightarrow a$.

We use truth table to find the solution:

a	B	A's statement, $b \Rightarrow a$	B's statement: $b \Rightarrow a$
T	T	T	T
T	F	T	T
F	T	F	F
F	F	T	T

We look for the row(s) where a 's value matches with A's statement and b 's value matches with B's statement. For this problem, the first row ($a=T, b=T$) is the only row that satisfies this condition. Therefore, we conclude that A and B are both truth-tellers.