Department of Computer and Information Sciences

CISC 1100, Structure for Computer Science

Practice Problem

1. [15 points, 5 points for each sub-problem] For each of the following sequence, write out the next two numbers in the sequence, and then write its recursive formula and closed formula.
   a. 1, 5, 9, 13, 17, ___, ___

Ans. Next two numbers are: 21, 25

Recursive formula is :

$a_1=1$

$a_n=a_{n-1}+4$

Closed formula is $a_n=4n-3$

   b. 2, 5, 8, 11, 14, ___, ____

Ans. Next two numbers are 17, 20

Recursive formula is :

$a_1=2$

$a_n=a_{n-1}+3$

Closed formula is $a_n=3n-1$

   c. 2, 6, 18, 54, 162, ___, ___

Ans. Next two numbers are 486, 1458

Recursive formula is :

$a_1=2$

$a_n=3a_{n-1}$

Closed formula is $a_n=2 * 3^{n-1}$

2. [5 points] Evaluate the following summations given in big sigma notations:

a. $\sum_{k=1}^{2}(2k-1)$

Ans. $\sum_{k=1}^{2}(2k-1)$ =1+3=4

b. $\sum_{k=-1}^{2}(k+1)$

Ans. $\sum_{k=-1}^{2}(k+1)$ =0+1+2+3=6.

3. [5 points] Express the following summations using the big sigma notations:

a. 2+3+4+5+6+7+8

Ans. 2+3+4+5+6+7+8= $\sum_{k=1}^{7}(k+1)$

b. $3+6+9+12+15$

Ans: $3+6+9+12+15=\sum_{k=1}^{5}3k$

4. [15 points, 3 points each] Let A={1,2}, B={4,6,8}, C={2}, and universal set U={1,2,3,4,5,6,7,8}. Answer the following questions on sets:

   a. A^c=__{3,4,5,6,7,8}________________________________
   b. $A \times B$ =_{(1,4), (1,6),(1,8), (2,4), (2,6), (2,8)}_________________
   c. P(B)=_{ {}, {4}, {6}, {8}, {4,6}, {4,8}, {6,8}, {4,6,8} }_______________
   d. $P(A \cap B) =$ _P({})= { {} }____________________
   e. $| P(A) \times B |=$_|P(A)| x |B| = $2^{|A|}$x |B| = 2^2x3=12___

5. [10 points, 5 points each] Interpret the following set builder notations, and find out all members of each set. Remembering that ***N*** stands for the set of natural numbers, i.e., 0, 1, 2, 3, …. ***Z*** stands for the set of integers, i.e., -4, -3, -2, -1, 0, 1,2,3, …. ***R*** stands for the set of real numbers; ***Q*** stands for the set of rational numbers, i.e. those numbers that can be written as fractions.
   a. $\{n : n \in Z \text{ and } n = k^2 \text{ for some integer k, k} > 0 \text{ and k} < 3\}$

      Ans. {1, 4}

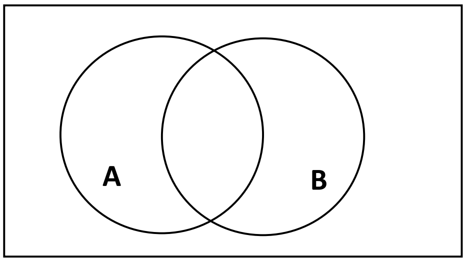
   b. $\{n^2 : n \in A\}$ where A={1,2,3}.

   Ans: {1, 4, 9}

6. [20 points, 10 points each] Reading, drawing and interpreting Venn Diagram.
   a. Draw a Venn Diagram, and shade the part that represents $A-(B \cap C)$. Suppose that A is the set of all Fordham students at Rose Hill campus, B is the set of all Fordham University freshman students, and C is the set of all Fordham University commute students. What does $A-(B \cap C)$ stand for (i.e., which body of students does this set refer to)?

Ans: we talked about it in class.

   b. Write a set expression to represent the shaded area in the following the Venn diagram. Suppose |A|=100, |B|=100, and $|A \cap B| = 20$, find out the cardinality of the set that the shaded area represents.



Ans; We talked about it in class.

7. [15 points, 5 points each] Draw truth table for the following propositions:
   a. $p \wedge q$

   Ans: see slides or book.

   b. $p \vee q$

Ans: see slides or textbook

   c. $p \Rightarrow q$

Ans: see slides or textbook

8. [15 points, 5 points each] Write truth table for the following propositions, and comment on whether each of them is a tautology.
   a. $\neg(p \Rightarrow q)$

   Sol.

p	q	$p \Rightarrow q$	$\neg(p \Rightarrow q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

It's not a tautology.

b. $(p \wedge q) \vee (\neg p \wedge \neg q)$

Sol.

P	q	$\neg p$	$\neg q$	$(p \wedge q)$	$(\neg p \wedge \neg q)$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
T	T	F	F	T	F	T
T	F	F	T	F	F	F
F	T	T	F	F	F	F
F	F	T	T	F	T	T

It's not a tautology.

c. $(p \wedge (p \Rightarrow q)) \Rightarrow q$

Sol.

P	q	$p \Rightarrow q$	$(p \wedge (p \Rightarrow q))$	$(p \wedge (p \Rightarrow q)) \Rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

It's a tautology.

9. [20 points] Draw truth tables for the following two pairs of propositions, and comment on whether they are equivalent to each other. Also explain the result using your intuition (one way to do this is to make up concrete examples, for example let p stands for "it rains", q stands for "it's cold", and interpret each of the propositions and see if they mean the same thing).
   a. $\neg(p \vee q)$ and $\neg p \vee \neg q$

Ans: I omit the truth tables here. Come to see me during office hour if you still need help writing truth tables.

After drawing truth tables for the two forms, we find they are not equivalent. This makes sense, as if we let p stands for "it rains", q stands for "it's cold", then:

- $\neg(p \vee q)$ means it's not the case that "it rains or it's cold". Here we negate the whole statement that "it rains or it's cold". This statement is true if it does not rain and it's not cold.
- $\neg p \vee \neg q$ means it does not rain or it's not cold.

b. $p \Rightarrow q$ and $\neg q \Rightarrow \neg p$

Ans.

P	q	$\neg p$	$\neg q$	$p \Rightarrow q$	$\neg q \Rightarrow \neg p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

We found that they are equivalent. This makes sense if we interpret the two forms by plugging meanings for p, q similar to a.

- $p \Rightarrow q$ means "If it rains, then it's cold".
- $\neg q \Rightarrow \neg p$ means "if it's not cold, then it does not rain".

They are both false when it rains, and it's not cold. (p=true, q=false). They are both true otherwise.

10. [10 points, 5 points each] For each of the following statements, introduce letters to represent the simple propositions and express the statements as a propositional forms:
    a. Healthy plant growth follows from sufficient water.

Ans: Let h stands for “healthy plant growth”, w stands for “sufficient water”. The statement can be represented as $w \Rightarrow h$.

    b. A sufficient condition for the knight to win is that the armor is strong or the horse is fresh.

Ans: Let w stands for “the knight wins”, s stands for “the armor is strong”, and h stands for “the horse is fresh”. The statement can be represented as $(s \vee h) \Rightarrow w$.

11. [10 points] Negate the forms that you wrote down for 10(a) and 10(b).
    10.a. $\neg(w \Rightarrow h) = \neg(\neg w \vee h) = (\neg\neg w) \wedge (\neg h) = w \wedge (\neg h)$
    10.b. $\neg((s \vee h) \Rightarrow w) = \neg(\neg(s \vee h) \vee w) = (\neg\neg(s \vee h)) \wedge (\neg w) = (s \vee h) \wedge (\neg w)$