CISC 1400— Discrete Structures
Fall, 2016

Homework Assignment #5

Some of these questions are from the following textbook, Discrete Mathematics with Applications, by Susanna S. Epp.

1 Answer the following questions:

(a) Define $F : \mathbf{N} \rightarrow \mathbf{N}$ by the rule $F(n) = 2 - 3n$, for all natural numbers $n \in \mathbf{N}$.

i. Is F a function? Why or why not?

ii. If we modify the codomain of F to be $\mathbf{Z}$ (the set of all integers), is F a function? If f is a function, is it an injective (i.e., one-to-one) function? Is it a surjective function? Briefly explain why.

(b) Define $G : \mathbf{R} \rightarrow \mathbf{R}$ by the rule $G(x) = 2 - 3x$ for all real numbers x. Is G onto? Explain your answer.

2 Use the space below to draw the so-called arrow diagrams for three functions that are 1) injective but not surjective, 2) not injective but surjective, 3) injective and surjective. You can choose the domain and codomain to be sets containing several elements of your choice.

3 If there are 400 freshman students in Fordham University, then there are at least two students who share the same birthday (i.e, they were born in the same day of the same month, even though not necessarily in the same year). Use Pigeonhole Theorem to explain, and describe what's the domain, codomain and function in applying Pigeonhole Theoreom in this example.

4 True or False. Explain briefly (you can use diagram to show, or one or two short sentences).

(a) For a function $f : X \to Y$, where both domain X and codomain Y are finite sets, if f is an one-to-one function, then $|X| \leq |Y|$.

(b) For a function $f : X \to Y$, where both domain X and codomain Y are finite sets, if f is an onto function, then $|X| \geq |Y|$.

5 (Extra Credits) It's possible to compose two functions to get a *composite* function. For example, let $f : \mathcal{R} \to \mathcal{R}$, and $f(x) = 2x + 1$ (i.e., function f maps a real number x to a real number given by $2x + 1$;), and let $g : \mathcal{R} \to \mathcal{R}$, and $g(x) = x^2$ (i.e., function g maps any real number to its square). Then function $(g \circ f)$ is defined as follows: $(g \circ f)(x) = g(f(x))$, i.e., function $(g \circ f)$ maps x to $g(f(x))$ (i.e., x is first mapped to a real number $f(x)$, and then $f(x)$ is in turn mapped to number $g(f(x))$ using rule of function g. So $(g \circ f)(x) = g(2x + 1) = (2x + 1)^2$.

i. Is $(g \circ f)(x)$ a function? Why?

ii. Calculate $(g \circ f)(1)$, $(g \circ f)(0.2)$.

iii. What is function $(f \circ g)(x) = f(g(x))$? Calculate $(f \circ g)(1)$, $(f \circ g)(0.1)$.