Homework Assignment #4 (Relation)

1 Consider the following rules on the set of natural numbers, $\mathcal{N}$, 1) write down 5 ordered pairs of natural numbers that are related under the relation, and 2) decide for the properties of reflexiveness (and irreflexiveness), symmetry (and antisymmetry) and transitivity. Note: please specify explicitly whether the relation has each of the five properties or not, and explain your answer.

(a) "is less than or equal to"

(b) "is a factor of"

(c) "is 3 less than"

2 Consider the following rules on the set of all Fordham students and examine the resulting relation for the properties of reflexiveness (and irreflexiveness), symmetry(and anti-symmetry) and transitivity.

   (a) "went to the same high school as" (assuming everyone went to high schools, and some might have gone to multiple high schools).

   (b) "share a same facebook friend with" (assuming every Fordham student is a Facebook user).

3 Examine each of the following relations, and decide whether they are reflexive, irreflexive, symmetric, anti-symmetric, and transtive.

(a) $R_1 = \{(x, y) \in \mathcal{N} \times \mathcal{N} : (x - y) \text{ is odd}\}$

(b) $R_2 = \{(x, y) \in \mathcal{N} \times \mathcal{N} : 2x \leq y\}$

(c) $R_3 = \{(a, b) \in \mathcal{N} \times \mathcal{N} : a \text{ and } b \text{ are coprime with each other (i.e., the greatest common divisor of a and b is 1}\}.$

4 Pick an online service that provides certain kinds of social networking service (such as Facebook, twitter, LinkIn, ancestry, ...) as your case studies, and perform some background research to find out the following:

(a) For the underlying relation(s) that the social networking service keeps track, how large is the user base (i.e., how many users does the service has)? (Please cite your source).

(b) What kind of discovery services does the site provide for its users? (e.g., Facebook suggests possible friends to its users). Make a guess at how it works.

(c) Could you suggest new services for the site? Or do you have any hypothesis about the network that is formed using the service (e.g., six-degree of separation)?