Introduction

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What's discrete mathematics ?

- Discrete mathematics: dealing with objects that can assume only distinct, separated values
  - Sequence, set
  - Logic
  - Relations, functions
  - Counting, probability
  - Graphs
- Useful for modeling real world objects
- Especially useful for computer problem solving

Discrete mathematics is concrete, i.e., very practical …

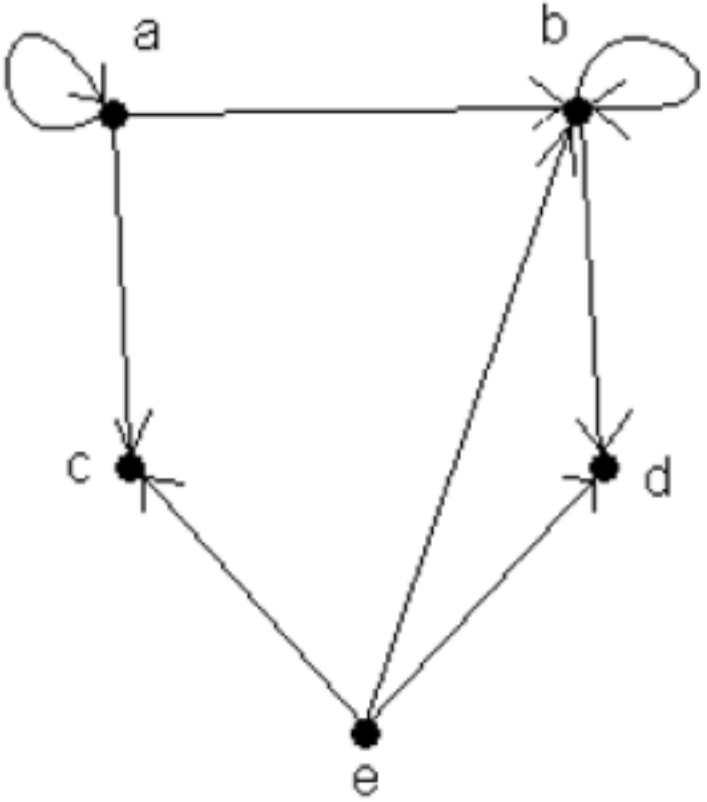
We start with set …

- Set is everywhere …
  - the group of all students in our class is a set
  - the group of all freshmen in our class is a set
- Some set are subset of another set
- Some sets are disjoint, i.e., have no common elements
  - e.g., the set of freshmen and the set of sophomore
- Operations on sets makes sense too
  - union, intersection, complement, …

With set, we define relations

- Among the set of all students in our class, some pairs are special …
  - The pairs have same birthday
  - The pairs are from same states
  - The first is older than the second
- All are binary relation defined on a set of students

Graph representation of relations



Graph is a way to visualize relations

- A graph for “having same birthday” relation among class members
- An airline graph represents “having direct flight” relation
- A network graph connects two nodes if they are connected (via a wire or a wireless radio).
- …

Graph problems

- Can you draw the following picture without lifting the pencil or retracing any part of the figure ?

Graph: many real world applications

- Computer network: how to send data (URL request you type in browser) from your PC to a web server ?
- Engineering: how to connect five cities with highway with minimum cost ?
- Scheduling: how to assign classes to classrooms so that minimal # of classrooms are used?
- …

Functions as a special type of relations…

- Where one element in a set is related (mapped) to one and only one element in another set
- "birthday of" can be viewed as a function defined on our set
  - Any student is mapped to the date when he/she was born

Our class: birthday remark

- Some says, “there are at least two students in the class that are born in the same month (not necessarily same year).”
- Do you agree ?
- Pigeonhole theorem
  - If put n pigeons into m holes, where n>m, there is at least a hole that has more than one pigeons.

Still too obvious ?

- Suppose I randomly pick some students from class, how many students do I need to pick to guarantee that there are at least two students of same gender among those I picked ?
  - Students: pigeons (x)
  - Gender: holes (2)
  - If x>2, then there are at least one gender that has more than one student
- Note: the tricky part is
  - Recognize the theorem/formula that applies
  - Map entities/functions in your problem to those in the theorem/formula

With set defined, one is naturally interested in its size, a.k.a. counting the number of elements in a set

Our class: counting problem

- Simple ones:
  - How many students are there in the class, i.e. the cardinality of the set ?
  - How many ways can we elect a representative ?
- How many ways can we elect a representative and a helper ?
- How many ways can we form studying groups of 2 students (3 students, …) ?

Counting problem: history

- First known results on counting goes back to six century BCE's India:
  - Using 6 different tastes, bitter, sour, salty, astringent, sweet, hot, one can make 63 different combinations…
- first formula for counting combinations appears more than one thousand years later

$$C(n,r) = \frac{n!}{(n-r)!r!}$$

- # of ways to elect two class representatives

$$C(14,2) = \frac{14!}{(14-2)!2!} = \frac{14\times 13}{2} = 91$$

Counting is essential for studying probability, i.e., how likely something happens …

Ex: Probability problems

- Suppose I choose one person randomly, what's the probability that you will be chosen ?
- Suppose I choose two persons randomly, what's the probability that you and your neighbor are chosen ?
- What's the probability of winning NY lottery ?

Logic: a tool for reasoning and proving

An example

- Your friend's comment:
  - If the birds are flying south and the leaves are turning, then it must be fall. Falls brings cold weather. The leaves are turning but the weather is not cold. Therefore the birds are not flying south.
- Do you agree with her ?
- Is her argument sound/valid?

An example

- Is her argument sound/valid?
  - Suppose the followings are true:
    - If the birds are flying south and the leaves are turning, the it must be fall.
    - Falls brings cold weather.
    - The leaves are turning but the weather is not cold.
  - Can one conclude “the birds are not flying south” ?

Reasoning & Proving

- Prove by contradiction
  - Assume the birds are flying south,
  - then since leaves are turning too, then it must be fall.
  - Falls bring cold weather, so it must be cold.
  - But it's actually not cold.
  - We have a contradiction, therefore our assumption that the birds are flying south is wrong.

So we have seen a list of topics …

- Sequence
- Set
- Logic
- Relation, Function
- Counting
- Probability
- Graph

Goals

- Master the basics of discrete mathematics
- Develop mathematical and computational reasoning abilities
- Become more comfortable and confident with both mathematics and computation

Discrete structure is essential for computer problem solving

Computer problem solving

- Model real world entity
  - Student records in a registration system=> objects in a set
  - Network nodes => graph vertices
- Develop/identify algorithm for solving specific problem
  - Search for a student record using name (or ID, …)
  - Query for a course using a prefix (all CSRU courses ?)
  - Find shortest path in a graph
- Implement algorithm using a programming language that computers "understand"

Computer projects

- We will learn basic web programming
  - Build your own web page
  - Learn HTML, JavaScript, …
- Use Alice to build 3D animation clip
  - Cartoon, simple game …

Let’s look at syllabus …

Expectations of students

- Think, think, think and practice
  - Make sense of the concepts, notations
  - Relate to your intuitions
  - Reflect about connections among different concepts
- Active participation in class
  - There are no silly questions !
- Keep up with homework
- Take advantage of office hour and tutor room