Relations & Functions

CISC1100, Spring 2013

Fordham Univ

Overview: relations & functions

- Binary relations
  - Defined as a set of ordered pairs
  - Graph representations
- Properties of relations
  - Reflexive, Irreflexive
  - Symmetric, Anti-symmetric
  - Transitive
- Definition of function
- Property of functions
  - one-to-one
  - onto
  - Pigeonhole principle
  - Inverse function
- Function composition

Relations between people

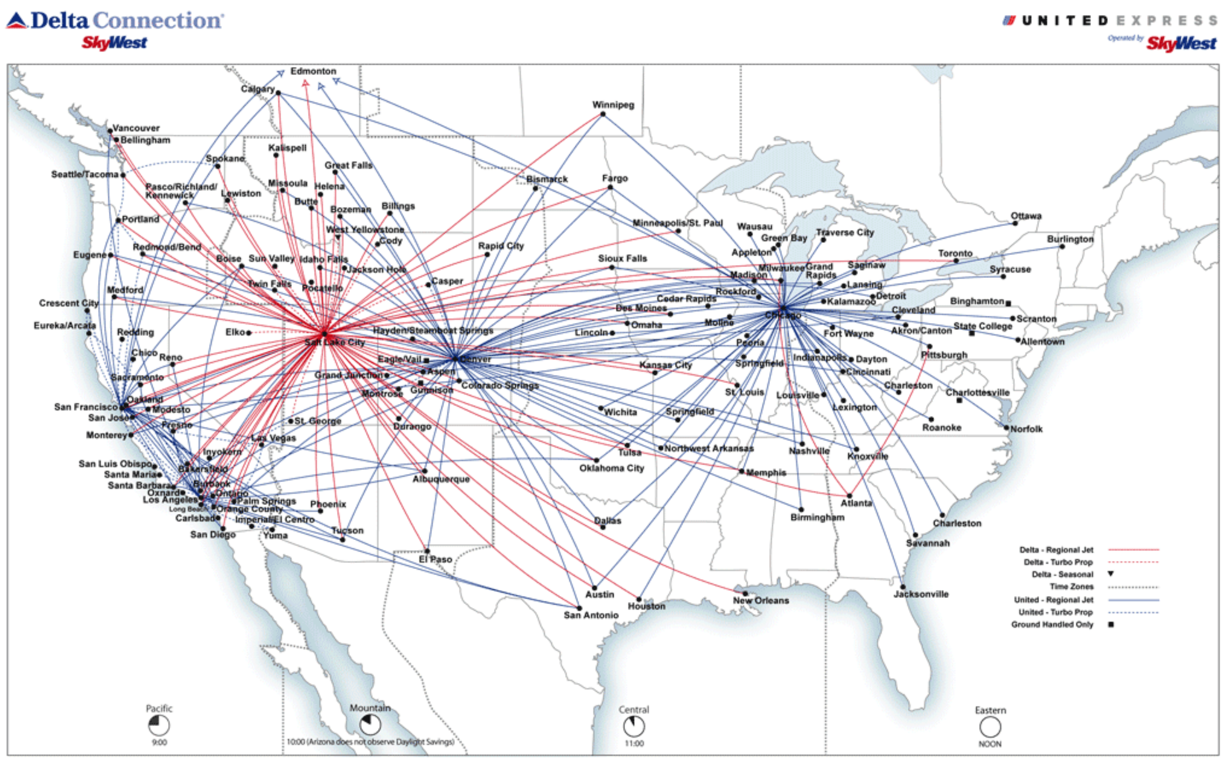
- Two people are related, if there is some family connection between them
- We study more general relations between two people:
  - "is the same major as" is a relation defined among all college students
    - If Jack is the same major as Mary, we say Jack is related to Mary under "is the same major as" relation
    - This relation goes both way, i.e., symmetric
  - "is older than" defined among a set of people
    - This relation does not go both way
  - " is facebook friend with", …

Relations between numbers

- Comparison relation
  - =, <, >, <=, …
- Other relations
  - Add up to 10, e.g., 2 and 8 is related under this relation, and so is 5 and 5, …
  - Is divisible by
    - a is divisible by b, if after dividing a by b, we get a remainder of 0
    - E.g. 6 is divisible by 2, 5 is not divisible by 2, 5 is divisible by 5, …

Relation is a graph

- nodes (solid small circle): cities,…
- Arcs: connecting two cities, … that are related (i.e., connected by a direct flight)
  - with Arrows: the direction of the "relation"…



Ex: Relations between sets

- Given some sets, {},{1}, {2}, {1,2}, {1,2,3}
  - "Is a subset of" relation:
    - {} is a subset of {1}
    - {1} is a subset of {1,2}, …
  - Practice: draw the graph for each of above relations
  - "Has more elements than" relation:
    - {1} has more elements than {}, …
  - "Have no common elements with" relation:
    - {} has no common elements with {1},
    - {1} has no common elements with {2}…

Binary relations: definition

- Relations is defined on a collection of people, numbers, sets, …
  - We refer to the set (of people, numbers, …) as the **domain** of the relation, denoted as S
  - A **rule** specifies the set of ordered pairs of objects in S that are related
    - Rule can be specified differently

Ways to describe the rule

- Consider domain S={1,2,3}, and "smaller than" relation, $R_<$
- Specify rule in English: "a is related to b, if a is smaller than b"
- List all pairs that are related
  - 1 is smaller than 2, 1 is smaller than 3, 2 is smaller than 3.
  - (1,2),(1,3),(2,3) are all ordered pairs of elements that are related under $R_<$
  - i.e., $R_<$={(1,2), (1,3),(2,3)}

Formal definition of binary relation

- For domain S, the set of all possible ordered pairs of elements from S is the Cartesian product, S x S.
- Def: a binary relation R defined on domain S is a subset of S x S
- For example: S={1,2,3}, below are relations on S
  - $R_1=\{(1,2)\}$
  - $R_2=\{\}$, no number is related to another number

$$R_3 = \{(a,b) \mid a \in S \text{ and } b \in S \text{ and } a+b > 2\}$$

Formal definition of binary relation(cont'd)

- Sometimes relation is between two different sets
  - "goes to college at" relation is defined from the set of people, to the set of colleges
- Given two sets S and T, a binary relation from S to T is a subset of SxT.
  - S is called domain of the relation
  - T is called codomain of the relation
- We focus on binary relation with same domain and codomain for now.

Domain can be infinite set

Domain: Z

R: {(a, b) is an element of Z x Z : (a - b) is even}

Given any pair of integers a, b, we can test if they are related under R by checking if a-b is even

e.g., as 5-3=2 is even, 5 is related to 3, or

$$(5,3) \in R$$

e.g., as 5-4 is odd,

$$(5,4) \notin R$$

Example

- For the following relation defined on set {1,2,3,4,5,6}, write set enumeration of the relation, and draw a graph representation:
  - R_d: "is divisible by": e.g., 6 is divisible by 2
  - R_d = {(1,1), (2,1), (3,1), (4,1), (5,1), (6,1), (4,2), (6,2), (6,3)}

Some exercises

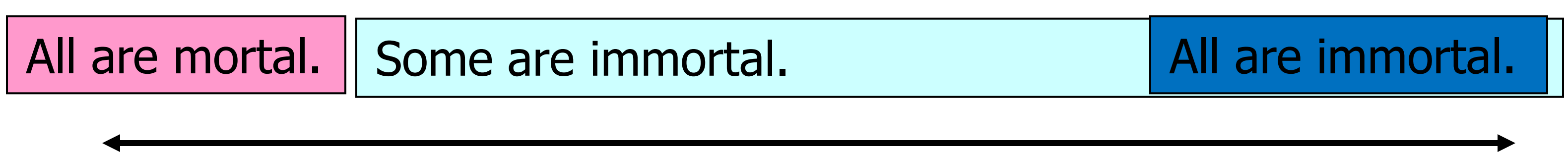
- For each of following relations defined on set {1,2,3,4,5,6}, write set enumeration of the relation, and draw a graph representation:
  - $R_{\leq}$:"smaller or equal to"
  - R_a: "adds up to 6", e.g., (3,3), (1,5) …

Relationships have properties

- Properties of relations:
  - Reflexive, irreflexive
  - Symmetric, Anti-symmetric
  - Transitive
- We will introduce the definition of each property and learn to test if a relation has the above properties

Primer about negation

- **Let's look at a statement that asserts something about all human being:**
  - All human beings are mortal.   (a)
- **The** opposite **of statement:**
  - All human beings are immortal.
- **The** negation **of statement ($\neg$a):**
  - It's not true that "all human beings are mortal"
  - i.e., Some human beings are not mortal.

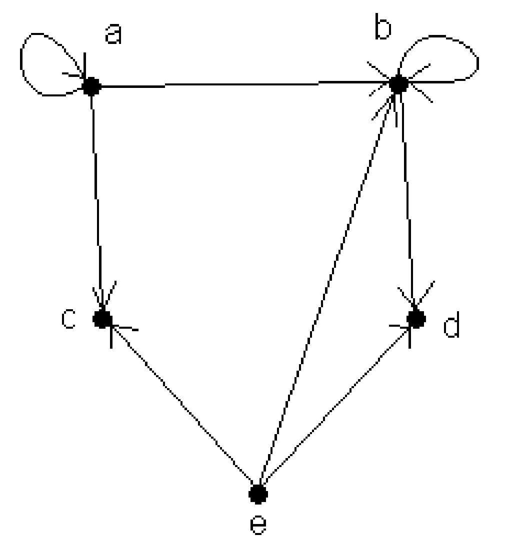


Reflexive Property

- Consider "is the same age as" relation defined on the set of all people
- Does this relation "go back to itself", i.e., is everybody related to himself or herself?
  - Tom is the same age as Tom
  - Carol is the same age as Carol
  - Sally is the same age as Sally
- For any person, he/she is the same age as himself or herself.
- The relation "is the same age as" is reflexive

Reflexive Property

- Def: A relation is reflexive if **every** element in the domain is related to itself
  - If R is reflexive, there is a loop on every node in its graph
- A relation is not reflexive if there is some element in the domain that is not related to itself
  - As long as you find one element in the domain that is not related to itself, the relation is not reflexive



Not reflexive since e does not go back to e

Try this mathematical one

- Domain is Z
- R={(x, y) | x,y$\in$ Z, and (x + y) is an even number}
- Is R reflexive?
  - Is any number in Z related to itself under R ?
  - Try a few numbers, 1, 2, 3, …
  - For any numbers in Z ?
  - Yes, since a number added to itself is always even (since 2 will be a factor), so R is reflexive

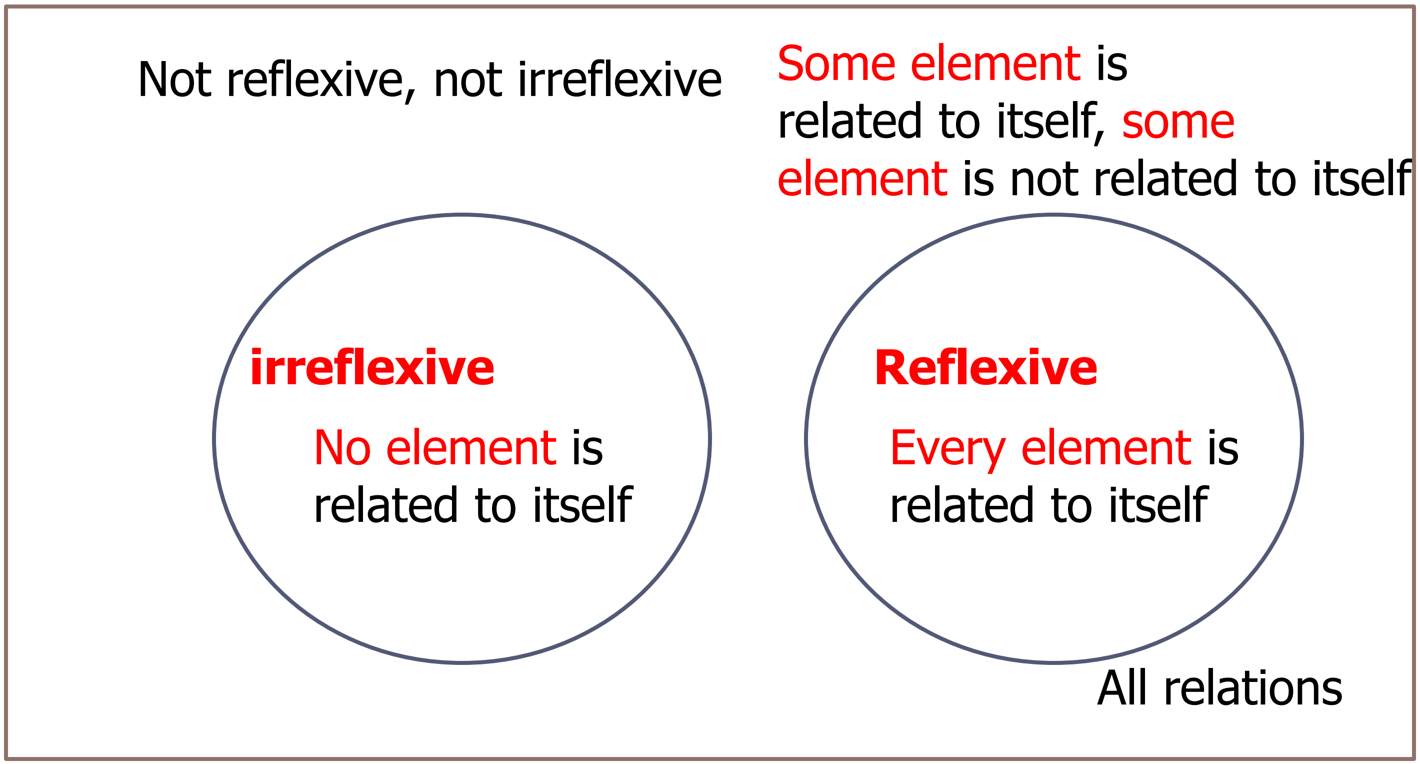
Another example

- Domain: R (the set of all real numbers)
- Relation: "is larger than"
- Try a few examples:
  - Pick a value 5 and ask "Is 5 larger than 5" ?
  - No, i.e., 5 is not related to itself
- Therefore, this relation is NOT Reflexive
- Actually, no real number is larger than itself
  - No element in R is related to itself, **irreflexive** relation

Irreflexive Relation

- For some relations, no element in the domain is related to itself.
  - "greater than" relation defined on R (set of all real numbers)
    - 1 is not related to itself under this relation, neither is 2 and 3 related to itself, …
  - "is older than" relation defined on a set of people
- Def: a relation R on domain A is **irreflexive** if every element in A is not related to itself
  - An irreflexive relation's graph has no self-loop

For all relations



A relation cannot be both reflexive and irreflexive.

reflexive? irreflexive ? Neither?

- Each of following relations is defined on set {1,2,3,4,5,6},
  - $R_{\leq}$:"smaller or equal to"
    - Reflexive, as every number is equal to itself
  - R_a: "adds up to 6", e.g., (3,3), (1,5) …
    - Neither reflexive (as 1 is not related to itself), or irreflexive (as 3 is related to itself)
  - R={(1,2),(3,4), (1,1)}

$$R = \{(a,b) \in Z \times Z : a^2 + b \text{ is odd}\}$$

Symmetric Property

- some relations are mutual, i.e., works both ways, we call them symmetric
- E.g., "has the same hair color as" relation among a set of people
  - Pick any two people, say A and B
  - If A has the same hair color as B, then of course B has the same hair color as A
  - Thus it is symmetric
- Other examples:
  - "is a friend of", "is the same age as", "goes to same college as"
- In the graphs of symmetric relations, arcs go both ways (with two arrows)

Exercise: symmetric or not

- Domain: {1, 2, 3 ,4}
- Relation={(1, 2), (1, 3), (4, 4), (4, 5), (3, 1), (5,4), (2, 1)}
- Yes, it is symmetric since
  - (1,2) and (2,1)
  - (1,3) and (3,1)
  - (4,5) and (5,4)

- Domain: Z (the set of integers)
- Relation: add up to an even number

A relation that is not symmetric

- "is older than" relation
  - If Sally is older than Tom, then Tom cannot be older than Sally
  - We found a pair Sally and Tom that relate in one direction, but not the other
  - Therefore, this relation is not symmetric.
- Actually, for "is older than" relation, it never works both way
  - For any two people, A and B, if A "is older than B", then B is not older than A.

Anti-symmetric Property

- Some relations never go both way
  - E.g. "is older than" relation among set of people
  - For any two persons, A and B, if A is older than B, then B is not older than A
  - i.e., the relation never goes two ways
- Such relations are called anti-symmetric relations
- In the graph, anti-symmetric relations do not have two-way arcs.

Formal Definition of Symmetric

A relation R defined on domain A is symmetric if <u>for all $a, b \in A$, if $(a,b) \in R$, then $(b,a) \in R$</u>

- To decide whether a given relation is symmetric, you check whether underlined statement is true
  - Underlined statement claim something for "**all**" a and b…
  - If you find one pair of a and b that makes it false, then the whole statement is false
    - i.e., if you find a and b, such that $(a,b) \in R$, and $(b,a) \notin R$, then R is not symmetric

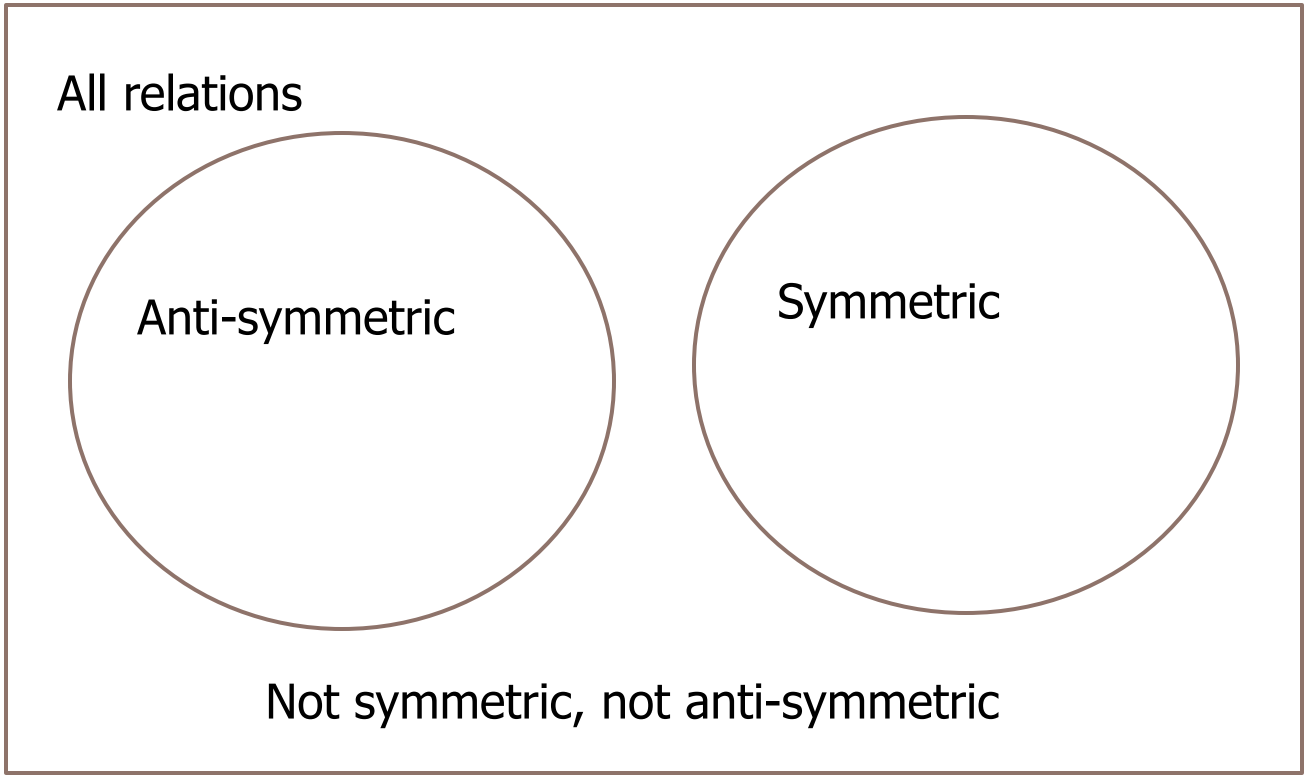
Formal Definition of Anti-symmetric

A relation R defined on domain A is anti-symmetric

if for all $a, b \in A$, if $a \neq b$ and $(a,b) \in R$ then $(b,a) \notin R$

- To decide whether a given relation is symmetric, you check whether underlined statement is true
  - Underlined statement claims something for "**all**" a and b…
  - If you find one pair of a and b that makes it false, then it is false
    - If you find a, b, a≠b, (a,b)∈R and (a,b)∈R, then R is not anti-symmetric

Symmetric & Anti-Symmetric



“Know the birthday of”

- Defined among our class of students
- For some pair of students, one knows the birthday of the other, but not vice versa
  - So the relation is not symmetric.
- Some pair of people know each other’s birthday
  - So this relation is not anti-symmetric.
- Therefore this relation is neither symmetric nor anti-symmetric

Exercises: Symmetric? Anti-symmetric ?

- For each of following relations defined on set {1,2,3,4,5,6}
  - R={(1,2),(3,4), (1,1),(2,1),(4,3)}
  - R={(1,2),(3,4),(1,1),(4,3)}
  - $R_{\leq}$:"smaller or equal to"
  - R_d: "divides": e.g., 6 divides 2

$$R = \{(a,b) \in Z \times Z : a^2 + b \text{ is odd}\}$$

Recall

- Reflexive, irreflexive property
  - Concerned about whether each object is related to itself or not
- Symmetric, anti-symmetric property
  - Concerned about each pair of objects that are related in one direction, whether they are related in another direction too.
- Next, transitive property
  - Concerned about every set of three objects …

Transitive: an introduction

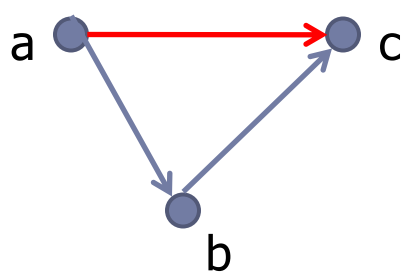
- You are assigned a job to draw graph that represents "is older than" relation defined on our class
  - You can only ask questions such as "Is Alice older than Bob?"
- Suppose you already find out:
  - Alice is older than bob
  - Bob is older than Cathy
  - Do you need to ask "Is Alice older than Cathy?"
    - No ! Alice for sure is older than Cathy.
- For any three people, a, b and c, if a is older than b, b is older than c, then fore sure, a is older than c.
- Such property of this relation is called transitive.

Is this relation transitive ?

- "is taking the same class as" relation on a set of students
- Suppose three students, Bob, Katie, and Alex,
  - Bob is taking the same class as Katie
  - Katie is taking the same class as Alex
- And now consider:
  - Is Bob is taking the same class as Alex ?
- Many cases: no
  - Bob takes 1400 with Katie, and Katie takes history with Alex, while Bob and Alex has no classes in common.
- Therefore this relation is not transitive

Transitive Property

- A relation R is transitive if for any three elements in the domain, a, b, and c, knowing that a is related to b, and b is related to c would allow us to infer that a is related to c.

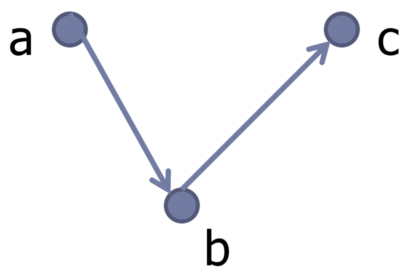


In graph of a transitive relation:
if there is two-hop paths from a to c,
then there is one-hop path from a to c.

- E.g. "is older than", "is same age as" is transitive

Not Transitive

- A relation R is not transitive if there exists three elements in the domain, a, b, and c, and a is related to b, b is related to c, but a is not related to c.



In graph: there is two-hop paths from a to c, but there is not a one-hop path from a to c.

- E.g. "is taking same class as", "know birthday of"

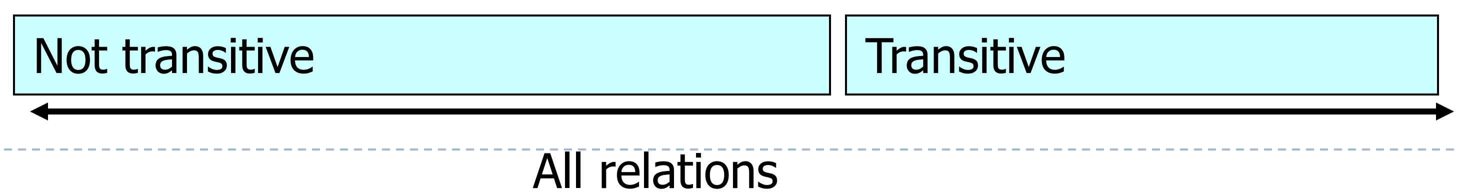
Formal Definition of Transitive

Relation R on domain A is transitive, if

for any a, b, c $\in$ A,

if $(a,b) \in R$ and $(b,c) \in R$,

then $(a,c) \in R$



Exercises: Transitive or not ?

- $R_{\leq}$:"smaller or equal to" defined on set {1,2,3,4,5,6}
  - For three numbers a, b, c from {1,2,3,4,5,6}
    - Would knowing that $a \leq b$, and $b \leq c$, allows me to conclude that $a \leq c$?
    - Yes !
  - It's transitive !
  - Let's check it's graph …

Exercises: Transitive or not ?

- Are the following relations defined on set {1,2,3,4,5,6} transitive ?
  - R_d: "is divisible by": e.g., 6 is divisible by 2
  - R_a: "adds up to 6", e.g., (3,3), (1,5) …

Example

- What properties does relation R has ?

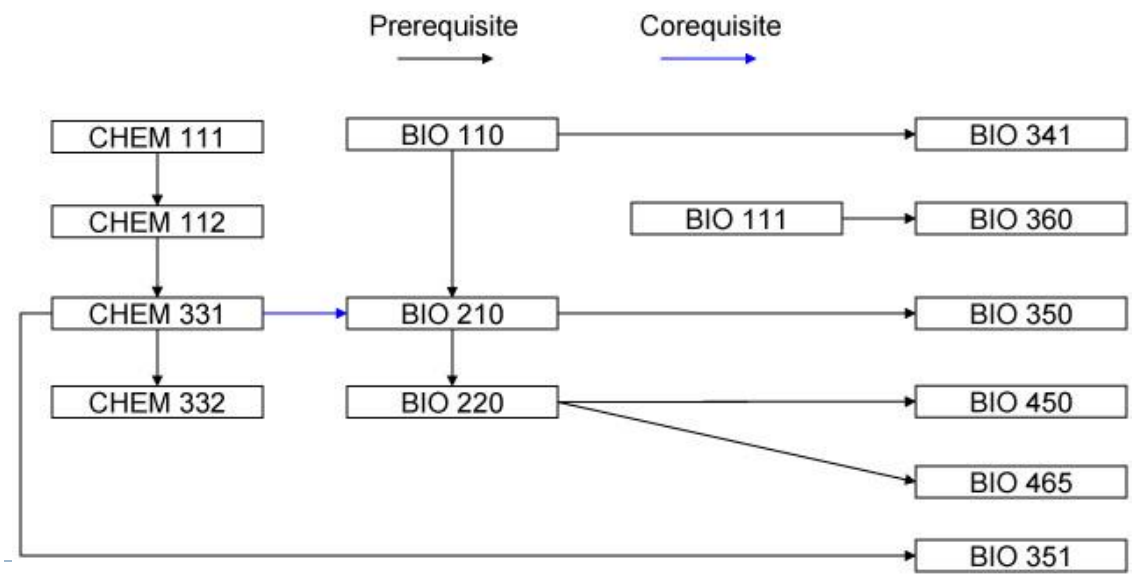
$$R = \{(a,b) \in Z \times Z : a^2 + b \text{ is odd}\}$$

Application: Partial Ordering

- A relation R on a set S that is reflexive, anti-symmetric, and transitive is called a **partial ordering** on S.
  - e.g. "less than or equal to"
  - e.g., "is a subset of",
  - e.g., "is prerequisite of"
  - If (a,b) is related under R, we call a is predecessor of b, and b is a successor of a.
- If furthermore, any two elements in domain is related (in one direction only), then it's a **total ordering**
  - E.g. less than or equal to
  - Is a subset of: is not a total ordering

Topological Sorting

- Topological sorting: given a partial ordering on S, order elements in S such that all predecessors appear before their successors.
  - e.g., Determine the order of taking courses based on prerequisite relation



The idea of algorithm

- Input: a list of ordered pairs each describing prerequisite requirement
  - e.g., (CS1, CS2): one needs to take CS1 before taking CS2
  - E.g. (CS2, Data Structure) …
- Output: an ordering of the courses, such that if (c1, c2) is in the prerequisite relation, then c1 appears before c2 in the ordering

Design the algorithm

- an **algorithm**: an effective method for solving a problem using a finite sequence of instructions.
- Step by step procedure to generate the output based on the input…
  - Work for different possible input
  - One algorithm should work for computer science student, biology student, physics student
- E.g. multiple digits addition multiplication

Finding the minimal numbers

- Problem setting
  - Input: a set of numbers $n_1, n_2, \ldots, n_k$
  - Output: the smallest number in the set
  - How would you do it?
- How to describe your approach so that other people (like programmers) can understand it ?

Finding the minimal numbers

- Problem setting
  - Input: a list of numbers $n_1, n_2, \ldots, n_k$
  - Output: the smallest number in the set
- Algorithm_Finding_Minimal

1. Set current minimal value to the first number in the set
2. Compare the next number from the list with the minimal value
3. if the number is smaller than the current minimal value
   then
   set the minimal value to the number
   endif
4. Repeat step 2,3 until reaching the last number in the list
5. Return the current minimal value

Functions

Functions are everywhere

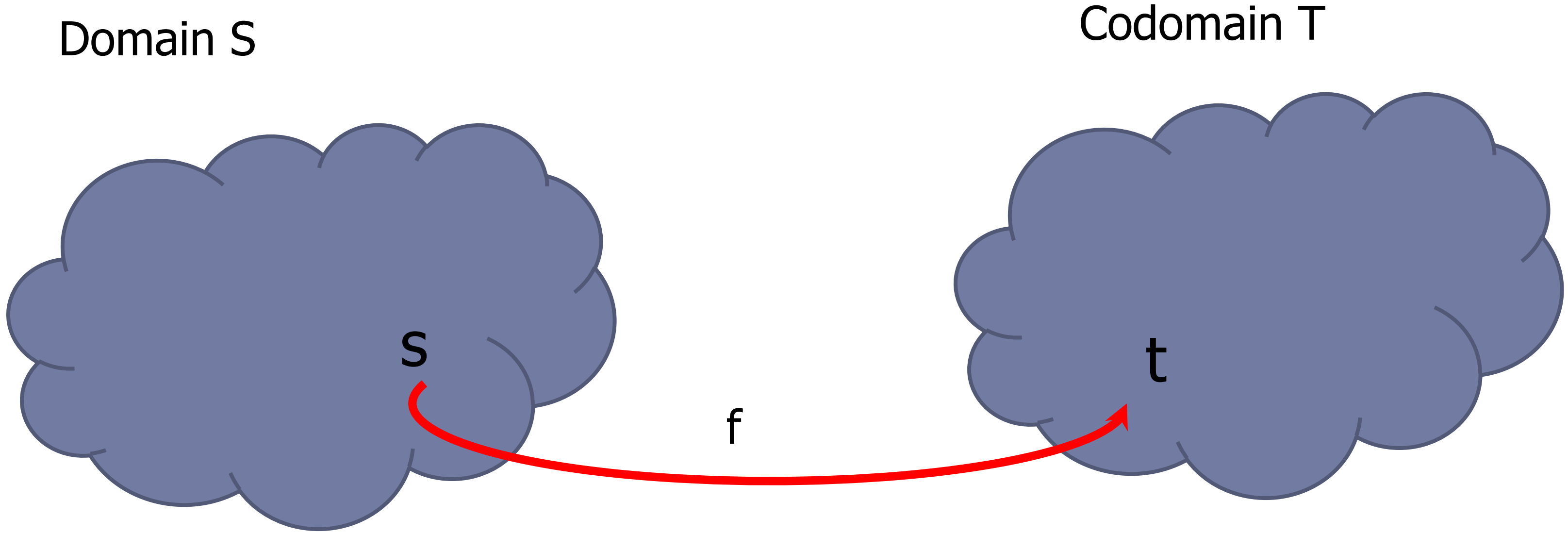
- A function is a way of transforming one set of things (usually numbers) into another set of things (also usually numbers).
- For example:
  - Fahrenheit to Celsius Conversion (link)
    - $[°C] = ([°F] − 32) \times \frac{5}{9}$
  - Closed formula of a sequence: maps position to value (link)
    - $a_n = 100*n+1$, $b_n = 2^n$

Components of a function

- Name, typically a letter like f, g, h, …
- Domain, a set of values
- Codomain, a set of values
- Rule: maps values in the domain to values in the codomain
  - For **every** value in the domain, the rule maps it into **a single value** in the codomain
- e.g. , f: $R \rightarrow R$, $f(x)=2x+1$

Function f : S→ T

Function f maps values in set S (domain) to values in set T (codomain)



f(s)=t
f maps s to t
t is called the image of s.
s is called the pre-image of t.

Useful analogy:
elements in S: pigeons
elements in T: holes
f(s)=t: pigeon s flies into hole t

Using mathematic formula

- For functions of numbers, the mapping can be specified using formula
  - f(a) = a + 4, “f of a equals a plus 4”
  - g(b) = b * b + 2, “g of b equals b times b plus 2”
  - h(c) = 5, “h of c equals 5”

Definition of function as relation

- Function f: A →B is a relation with domain A and codomain B, and for every $x \in A$, there is exactly one element $y \in B$ for which $(x, y) \in f$, we write it as f(x)=y.
  - i.e., a function is a relation where every element in the domain, say x, is related to exactly one element in the codomain, say y.

Useful analogy:
elements in S: pigeons
elements in T: holes
f(s)=t: pigeon s flies into hole t
every pigeon goes to one hole

Domain = Z (all integers) Codomain = Z (all integers)

$f(x) = x + 5$

- Ok, begin the same way (take values from the domain and put them in the formula)
- Choose 0. f(x) = 5 … it's in the Codomain
- Choose 1. f(1) = 6 … it's in the Codomain
- Choose -1. f(-1) = 4 ... it's in the Codomain
- But we can't do this forever
- Hand-waving argument
  - "Regardless of what integer I take from the domain, I can add 5 to that number and still have a value in the codomain."

Dealing with infinite sets

- If the domain is infinite, you can't try all values in the domain
- So you need to **look for values that might not work and try those.**
- If you can't find any domain values that don't work, **can you make an argument that all the domain values do work** ?

Domain = N (natural numbers, i.e., 0,1,2,…)

Codomain = N

f(x) = x-1

- Choose some values
  - Choose 0: f(0) = -1 … -1 is not in the codomain, it doesn't work
- so it's not a function

Functions with multiple variables

- Example:
  - f(x,y)=x-y, where x takes integer value, and y takes integer value
  - f maps an ordered pair of integers, i.e., x and y, to their difference (x-y), which is also an integer
- What's the domain ?
  - The set of ordered pairs of integers …
  - In mathematical notation: $Z \times Z$, the Cartesian product of Z

Outline

- Definition of function
- Property of functions
  - one-to-one
  - onto
  - Pigeonhole principle
  - Inverse function
- Function composition

Properties of Functions

- Two interesting properties of functions
  - **one-to-one**
  - **onto**
  - Bijective: one-to-one and onto

One-to-one function

- function f: S→T is one-to-one, if no two different values in the domain are mapped to the same value in codomain.
  - for two elements $s_1, s_2 \in S$, if $s_1 \neq s_2$, then $f(s_1) \neq f(s_2)$
  - Equivalently, for two elements $s_1, s_2 \in S$, if $f(s_1)=f(s_2)$, then $s_1=s_2$

Useful analogy:
elements in S: pigeons
elements in T: holes
f(s)=t: pigeon s flies into hole t
every pigeon goes to one hole
one-to-one function: no two pigeons go to same hole

Example

Domain = {1, 2, 3} Codomain = {1, 2, 3, 4} f(x) = x + 1

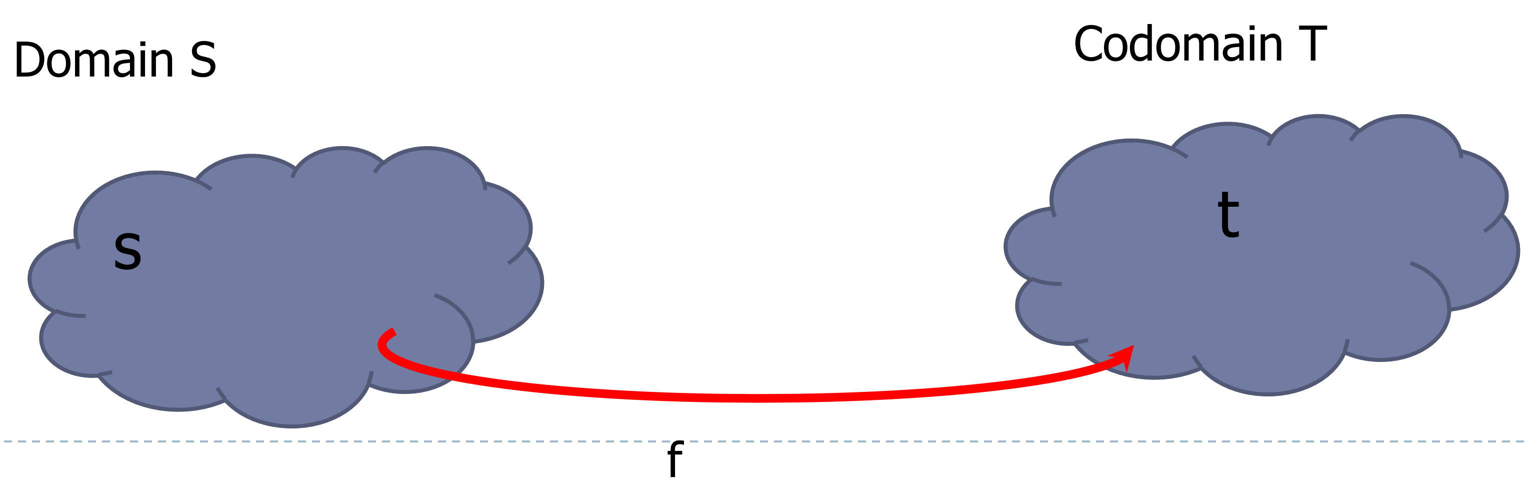
- So we must ask if it is a function?
  - Choose 1: f(1) = 2
  - Choose 2: f(2) = 3
  - Choose 3: f(3) = 4
  - So it is a function.
- Is it injective (one-to-one)? Well
  - we only reached value 2 by using x = 1.
  - we only reached value 3 by using x = 2.
  - we only reached value 4 by using x = 3.
- So it is one-to-one

Domain = {-2, -1, 0, 1, 2} Codomain = {0, 1, 2, 3, 4, 5, 6} $f(x) = x^2$

- Is it a function?
  - $f(-2) = 4, f(-1) = 1, f(0) = 0, f(1) = 1, f(2)=4$
  - So it is a function
- Is it one-to-one?
  - No. We can reach the value 4 in two ways.
  - $f(-2)=4$ and $f(2)=4$
- For any injective function $f: S \rightarrow T$, where S and T are finite, what kind of relation hold between $|T|, |S|$?

Pigeonhole Theorem

- Consider function f: S→T, where S, T are finite sets
  - If f is one-to-one, then |S|≤|T|
  - If |S|>|T|, then f is not injective.
    - at least two diff. values in S are mapped to same value in T
- Pigeonhole Theorem



Dealing with infinite sets

f: R→R with the rule $f(x)=x^2+4x+1$

- Is the function injective (one-to-one) ?
- What does this function look like ?
  - One can use Excel to plot a function (link)
- A function is injective if its graph is never intersected by a horizontal line more than once.

Onto Functions

- A function is onto if every value in the codomain is the image of some value in the domain (i.e., every value in the codomain is taken)
  - f: S→T is onto if for any element t in T, there exists an element s in S, such that f(s)=t
- Let Range of f, ran(f), be the set of all values that f can take:

$$ran(f) = \{f(x) : x \in S\}$$

  - For onto function f, ran(f)=T

Surjective function example

Domain = {1, 2, 3, 4} Codomain = {11, 12, 13, 14} f(x) = x + 10

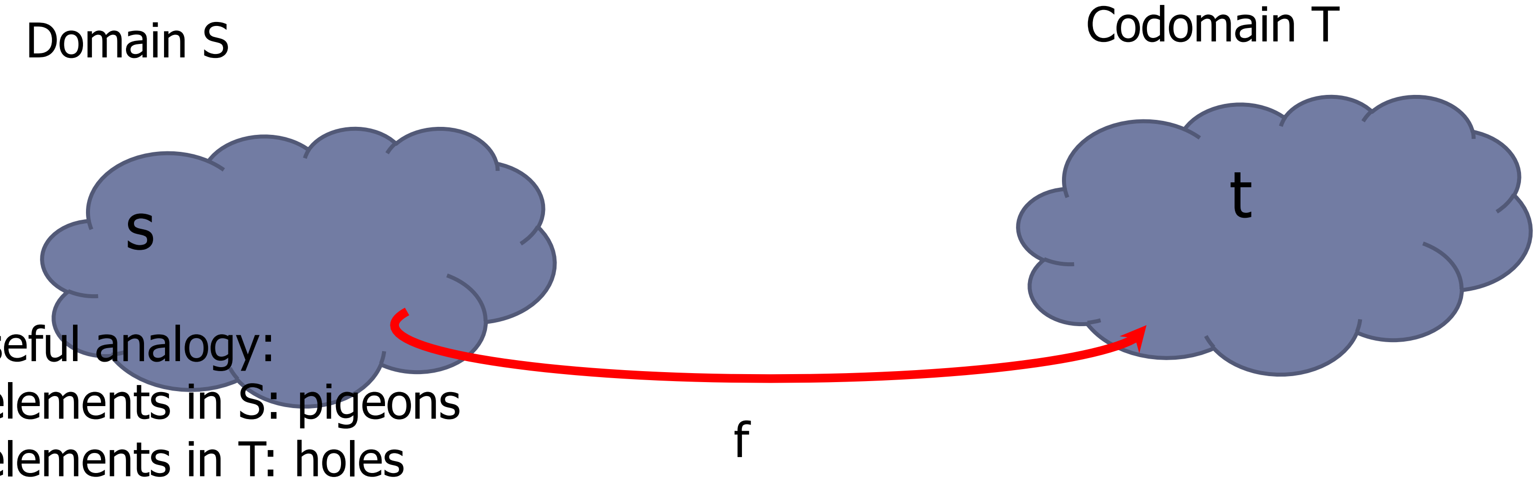
- First you have to figure out if it is a function or not.
  - Choose 1: f(1) = 11
  - Choose 2: f(2) = 12
  - Choose 3: f(3) = 13
  - Choose 4: f(4) = 14
  - So it is a function
- Is it onto?
  - Yes. Because we covered all of the Codomain values, i.e., every value of codomain is an image of some values in the domain.

Domain = {1, 2, 3} Codomain = {0, 1, 2, 3} $f(x) = x - 1$

- Determine whether it is a function
  - Choose 1: $f(1) = 0$
  - Choose 2: $f(2) = 1$
  - Choose 3: $f(3) = 2$
  - So it is a function.
- Is it onto?
  - No, we never arrived at the value 3 which is in the Codomain

Property of onto function

- Consider function f: $S \rightarrow T$, with S, T finite
  - If f is onto, then $|S| \geq |T|$
  - If $|S| < |T|$, then f is not onto
    - There is some element in T that is not mapped to



Useful analogy:
elements in S: pigeons
elements in T: holes
f(s)=t: pigeon s flies into hole t
every pigeon goes to one hole
onto function: every hole is occupied by some pigeon(s)

Dealing with infinite sets*

f: R→R with the rule $f(x)=x^2+1$

- Is it a function?
- Is it an onto function ?
  - For every

$$t \in \mathrm{R}, \text{ there exists some } s \in \mathrm{R}$$

$$\text{such that } f(s) = s^2 + 1 = t \;?$$

Bijection

- A function that is both onto and one-to-one is called a **bijection,** or we say the function is **bijective.**
- Consider function f: S→T, with S, T finite
  - If f is bijective (one-to-one and onto), then |S|=|T|
    - as f is one-to-one, we have |S|≤|T|
    - as f is onto, we have |S|≥|T|

Useful analogy:
elements in S: pigeons
elements in T: holes
f(s)=t: pigeon s flies into hole t
every pigeon goes to one hole
bijective function: every hole is occupied by exactly one pigeon

Invertible functions

- Give a function f: $S \to T$, what happens if we "invert" the mapping, and get a new relation ?
  - Make T the new domain
  - Make S the new codomain
  - If $s \in S$ is mapped to $t \in T$, we now map t to s
- Do we get a function this way ?
  - i.e., is any value in the new domain (T) being mapped to one and only one value in the new domain S ?
- If we get a function after we insert the mapping, the f is called invertible.

Formal definition of inverse

- Function $f: A \to B$ is invertible if there is a function $f^{-1}: B \to A$ such that $f(x)=y$ if and only if $f^{-1}(y)=x$.
- f^{-1}: read as "f inverse"

What kind of function is invertible?

- Every value in the codomain has values in the domain mapped to it
  - The function is onto.
  - <draw a diagram showing a function that is not onto, what happens when reverse mapping…>
- Every value in the codomain has only one value in the domain mapped to it.
  - i.e. the function is one-to-one
  - <draw a diagram of a function that is not one-to-one, what happens if reverse mapping?>
- A function is invertible if it's bijective.

Finding inverse function

- To find inverse of function f
  - First check if f is invertible (i.e., bijective)
  - Make the old codomain the new domain
  - Make the old domain the new codomain
  - Reverse the mapping

Reverse the mapping

- If the mapping is given by the set of ordered pairs
  - Just reverse the first- and second- components of each pair
- If the function is given by a diagram
  - Reverse the directions of the arrows
- If the function is given by a formula, f(x)
  - Solve the formula for x, i.e., express x in terms of f(x)

Inverse Example, f^{-1}

Domain = {2, 4, 6, 8} Codomain = {4, 8, 12, 16} f(x) = 2x

- New domain = {4, 8, 12, 16}
- New Codomain = {2, 4, 6, 8}
- Original mapping maps x to y=f(x)=2x
- Reverse mapping map y to x, i.e., given y, what's the x ? (express x in terms of y)
  - Solve y=2x for x, we get x=y/2.
- Inverse function is
  - $f^{-1}: \{4,8,12,16\} \rightarrow \{2,4,6,8\}$, $f^{-1}(y)=y/2$

Examples

- **Are the following functions invertible ? Find inverse for those invertible.**
  - f: R→R, with f(x)=3x+6
  - f:R →R, with f(x)=x^2
  - g:Z →Z, with the rule:

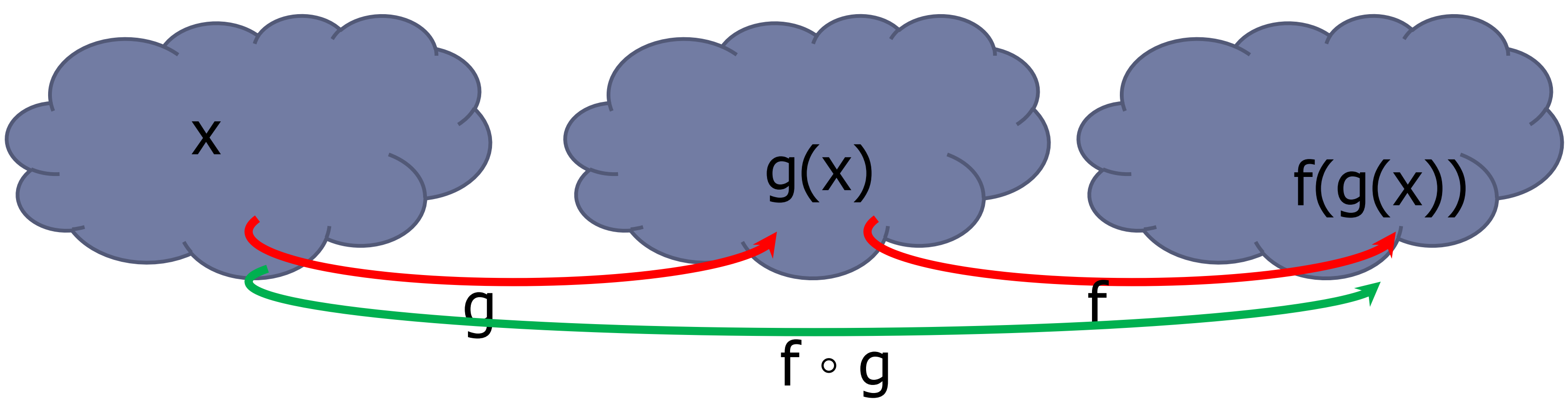
$$g(z) = -2z, \quad \text{if } z \le 0$$
$$g(z) = 2z - 1, \ \text{if } z > 0$$

Outline

- Definition of function
- Property of functions
  - Onto
  - One-to-one
  - Pigeonhole principle
  - Inverse function
- Function composition

Function Composition

- We can chain functions together – this is called composition (apply the mappings subsequently)
- $f \circ g$ (reads "f composed with g") defined as $f \circ g(x)=f(g(x))$ (note apply g first, then f)
  - First apply mapping of g, then apply mapping of f



Function Composition Example

- Example: for f, g with domain/codomain of R
  - $f(x) = x+5$, $g(x) = 2x + 3$
  - $f \circ g\ (x)=f(g(x))$

    $=f(2x+3)$

    $=(2x+3) + 5$

    $=2x + 8$

  - $g\circ f\ (x)=g(f(x))$

    $=g(x+5)$

    $= 2(x+5) + 3$

    $= 2x + 13$

Assume we have two functions with Domains and Codomains over all integers

$f(x) = 3x - 2$

$g(x) = x * x$

What is $f \circ g$?

$f(x*x) = 3(x*x)-2 = 3x^2-2$

What is $g \circ f$?

$g(3x-2) = (3x-2)*(3x-2)=9x^2-12x+4$

What is $f \circ f$?

$f(3x-2) = 3(3x-2)-2 = 9x-8$

What is $g \circ g$?

$g(x*x) = (x*x) * (x*x) = x^4$

What is $f \circ g$ for $g(2)$?

$f(g(2)) = f(4) = 3(4) - 2 = 10$

Function Compositions

Assume we have two functions with Domains and Codomains over all real numbers …

$f(x) = 3x - 2$

$g(x) = x^3$

$h(x) = x/4$

What is f^{-1}?

What is $f \circ f^{-1}$?

What is $f^{-1} \circ f$?

What is $f \circ g \circ f^{-1}$?

What is $f \circ f^{-1} \circ g$?