1.6   Exercises

1.6.1 Let $T = \{3, 5, 7, 9, 11, 22, 44\}$. Which of the following statements
are true?

(a) $7 \in T$

(b) $22 \in T$

(c) $T \in T$

(d) $11 \in T$

(e) $\{3, 5, 44\} \in T$

(f) $\{3, 5, 44\} \subseteq T$

(g) $\{3, 5, 7\} \in T$

(h) $\{3, 5, 7\} \subset T$

(i) $T \subset T$

1.6.2 Give an explicit listing for each of the following sets as defined by
the corresponding set builder notation:

(a) $\{x : x \in \mathbb{N} \text{ and } x^2 < 64\}$

(b) $\{x \in \mathbb{Z} : x^2 < 64\}$

(c) $\{3x : x \in \mathbb{Z} \text{ and } x \leq 5\}$

(d) $\{3x : x \in \mathbb{N} \text{ and } x \leq 5\}$

(e) $\{x : x = 2y + 3 \text{ and } y \in \{1, 2, 3, 4, 5, 6\}\}$

1.6.3 State the set builder notation that defines the set $A = \{4, 8, 16, 32, 64$

1.6.4 Given the following sets:

$$A = \{x \mid x \in \mathbb{Z} \text{ and } -4 < x \leq 7\}$$
$$B = \{y \in \mathbb{N} \mid y < 9\}$$
$$C = \{x \mid x = y + 3 \text{ for some } y \in \mathbb{Z} \text{ such that } -6 \leq y \leq 6\}$$
$$D = \{x \mid x \in \mathbb{Z} \text{ and } 3 \leq x \leq 9 \text{ and x is odd}\}$$
$$E = \{x \in \mathbb{Z} : -5 \leq x \leq 5\}$$
$$F = \{6, 3, 7, 4, 1, 0\}$$

Determine the following:

(a) $(C \cap D) \cup (F \cap E)$

(b) $(A - C)$

(c) $(C - A) \times (F \cap D)$

(d) $\mathscr{P}(B \cap D)$

(e) $|(F - D) \cap E|$

1.6.5 Given the following sets:

$$A = \{0, 2, 4, 6, 8, 10\}$$
$$B = \{1, 3, 5, 7, 9\}$$
$$C = \mathbb{Z}$$
$$D = \emptyset$$
$$E = \{ x \mid x \in C \text{ and } x > 0 \text{ and } x \text{ is even } \}$$
$$F = \{ x \in C \mid x > 0 \text{ and } x \text{ is odd } \}$$
$$G = \{1, 2\}$$

Determine the following:

(a) $A \cap C$

(b) $B - F$

(c) $B - (E \cap F)$

(d) $(A \cup B) \cap E$

(e) $(A \cup B) - C$

(f) $(B \times G) \cup (B \times G)$

(g) $\mathscr{P}(G)$

(h) $|\mathscr{P}(A)|$

(i) $\mathbb{Z} - ((E \cup F) \cup C \cap D))$

1.6.6 From a group of people asked at random which foreign languages they spoke fluently,

- 32 answered they spoke Spanish fluently,

- 18 answered they spoke Italian fluently and

- 9 answered they spoke Spanish and Italian fluently.

    Hint: Use a Venn diagram.

(a) How many people are fluent in Italian but not Spanish?

(b) How many people are fluent in Spanish but not Italian?

(c) How many people are fluent in Spanish or Italian?
    Hint: Use the principle of inclusion/exclusion.

1.6.7 A freshman at XYZ University has selected the courses to take during his or her first semester freshman year. Given the following sets:

T = {English Literature, Calculus, Biology, Philosophy}

I = {Philosophy, Journalism, Calculus}

C = {English Literature, History, Philosophy, Calculus, Psychology,
     Spanish, Biology, Visual Arts}

where T represents the set of courses that the student is currently taking, I represents the set of courses the student is interested in taking and C represents the set of courses available.

Use set operators to represent each of the following statements. In addition, give an explicit listing of the resulting set:

(a) The set of courses the student is taking and is interested in.

(b) The set of courses the student is taking or is interested in.

(c) The set of courses the student is taking but not interested in.

(d) The set of courses the student is interested in and still needs to take.

(e) The set of possible courses the student can choose from for next semester.

1.5.8 Let

$X = \{hat\},$

$Y = \{hat, \{gloves\}, \{\{hat,gloves\}\}, \{hat, \{gloves\}\}\},$

$Z = \{\emptyset, \{\{hat\}\}, \{hat, \{gloves\}\}\}.$

(a) What is $|X| + |Y| + |Z|$?

(b) Does $|X| + |Y| + |Z| = |X \cup Y \cup Z|$?

(c) Is $X \subseteq Y$?

(d) Is $X \in Y$?

(e) Is hat $\subseteq Y$?

(f) Is $\{\{hat, \{gloves\}\}\} \subseteq Y$?

(g) Is $X \subseteq Z$?

1.5.9 Explain why $|\mathscr{P}(X)| \neq |X|$ for any finite set X.

1.5.10 Is there a set X such that $|\mathscr{P}(X)| = 3$? If so, describe the set X; if not, explain why.

1.5.11 If the $|X| = 0$, how many elements would be in its power set?

1.5.12 Given two sets X and Y, which will grow faster as $|X|$ and $|Y|$ increase: $\mathscr{P}(X \times Y)$ or $\mathscr{P}(X) \times \mathscr{P}(Y)$? Can they ever be the same? Explain your answers.

1.5.13 Suppose that

$$A = \{c, o, m, p, u, t, e, r, s, i, n\}$$
$$B = \{m, a, t, h, e, i, c, s\}$$
$$C = \{c, h, e, m, i, s, t, r, y\}$$
$$D = \{e, n, g, l, i, s, h, t, r, a, u\}$$
$$U = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$$

Determine the following:

(a) $A \cup B$

(b) $A \cap B$

(c) $A \cap B \cap C$

(d) $(D \cup A) - B$

(e) $(A - B) \cup (B - A)$

(f) $(A - B) \cap (B - A)$

(g) $(A \cap C) \cup (B \cap D)$

(h) $A \cap (C \cup D)'$

(i) $((A \cup B) \cap (C \cup D))'$

(j) $(A \cup B \cup C \cup D)'$

(k) $|\mathscr{P}(D)|$

(l) $|A \times B|$

1.6.14 Given two sets A and B with $|A| = 9$ and $|B| = 17$. Is it necessarily true that $|A \cup B| = 26$? Explain your answer.

1.6.15 Given two sets X and Y assume that the $|Y| = 20, |X \cup Y| = 50$ and $|X \cap Y| = 10$. What is $|X|$?

1.6.16 [SN] Suppose that Grace, the most connected user of our social network (on page 317), is creeping around one evening trying to determine who likes what types of music. Of the friends whose profiles she is able to access, she determines that

- Sandra, Alyssa, Lauren and Ellen like classic rock, country, jazz and hip-hop.

- Manny, Chrissy, Niko, and Rania only like jazz.

- Peter likes just classic rock and

- Angela and Luka only like country.

  Let

- R be the set of friends who like classic rock,

- C be the set of friends who like country,

- *J* be the set of friends who like jazz, and

- *H* be the set of friends who like hip-hop.

Answer the following questions, then answer the last three questions using set operators:

(a) Is $H \subseteq R$?

(b) Is $C \subseteq R$?

(c) How many friends like classic rock and country?

(d) Use set notation to show how many friends like classic rock or jazz?

(e) Is the set notation that represents the how many friends who like classic rock and jazz the same?

(f) Use set notation to represent the set of friends who like jazz, but not classic rock.

1.6.17 Let the universal set U be comprised of the set of all students of a university in New York and let

- *S* be the set of students receiving academic scholarships,

- *L* be the set of students receiving student loans, and

- *W* be the set of students who will be working at least fifteen hours per week during the school year.

Use set operators to represent each of the following. In addition draw a Venn diagram and shade in the appropriate region. Note you can assume that if a student is working, he or she is working at least 15 hours a week.

(a) The set of students who are working and receiving both scholarships and loans.

(b) The set of students who are working, but not receiving any financial assistance.

(c) The set of students who are working or receiving student loans.

(d) The set of students who are working and receiving scholarships, but not loans.

1.6.18 Following the same sets specified in Problem 1.6.17, additionally let

- F be the set of female students of this University.

  Use set operators to represent each of the following:

(a) The set of all male students.

(b) The set of all female students who are working at least 15 hours per week.

(c) The set of all male students not receiving any financial assistance.

(d) The set of all female students receiving student loans.

(e) The set of all male students receiving scholarships or loans.

1.6.19 Given some set A and some universal set U, determine the following sets (in terms of A). *Note that the answer does not depend on the specific elements of set A and can be answered in general terms.*

   Example: $A \cap A' = \emptyset$

(a) $A - A'$

(b) $(A')'$

(c) $U - A$

(d) $A \cup A'$

(e) $(A \cap A) - (A \cup A)$

1.6.20 Sixteen people responded to an e-mail survey on family pets. Of the 16 who responded, 9 stated they had hamsters, 7 stated they had guinea pigs and 4 stated they had neither hamsters or guinea pigs. Use the principle of inclusion/exclusion to determine how many had both hamsters and guinea pigs.

   Hint: Use a Venn diagram.

1.21 Let X and Y be sets. Is it possible for $X \subset Y$ and $Y \subset X$? Explain why or why not.

1.22 [SN] Using our social network diagram (on page 317) and the set of attributes representing the basic information as listed for each friend in Table A.1 (on page 318), let

$$N = \{\text{attributes for Niko}\},$$
$$S = \{\text{attributes for Sam}\},$$
$$F = \{\text{attributes for Frank}\},$$
$$L = \{\text{attributes for Luka}\},$$
$$C = \{\text{all colleges/universities in the U.S.}\}.$$

Use set operators to represent each of the following:

(a) All the attributes that these four friends have in common.

(b) How many attributes these four friends have in common.

(c) The set of attributes that Niko and Sam have in common or the set of attributes that Frank and Luka have in common.

(d) The set of attributes that are unique to Niko.

(e) The set of colleges attended by these four friends.

1.23 [SN] Using our social network graph as represented (on page 317) and the set of attributes representing the basic information as listed for each friend in Table A.1 (on page 318), let

$$X = \{\text{Larry's friends}\}$$
$$Y = \{\text{Peter's friends}\}$$
$$P = \{\text{attributes for Peter}\}$$
$$L = \{\text{attributes for Larry}\}$$
$$F = \{\text{all members of the network}\}$$
$$C = \{\text{all cities in the U.S.}\}$$

Use set operators to represent each of the following:

(a) Set of friends of Larry or Peter.

(b) Set of Larry's friends who are not Peter's friends.