Example questions:

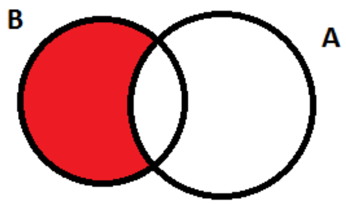
First, let us group Alice characters and objects into sets:

U = universal set of all Alice characters and objects
A={astronaut, T-Rex, AliceLiddell, hare, wizard, skeleton}
B={gazebo, cabin, sailboat, volcano}
C={AliceLiddell, hare, cat}
D={wizard, fairy, dragon}

What are the contents for each new set calculated below? When there is a *(V)* at the front of the question, draw a Venn diagram as well.

(V) B-A

{gazebo, cabin, sailboat, volcano}



P(D)

CxB

{(AliceLiddell,gazebo), (AliceLiddell,cabin), (AliceLiddell,sailboat), (AliceLiddell,volcano), (hare,gazebo), (hare,cabin), (hare,sailboat), (hare,volcano), (cat,gazebo), (cat,cabin), (cat,sailboat), (cat,volcano)}

(V) C∩A

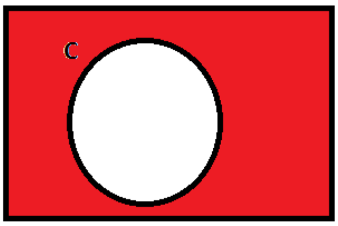
(V) (B∪D)-C

P(A∩B)

Draw Venn diagrams for the following. You do not need to list the contents of the resulting sets:

(B∪C)∩A

C'



B∩D'

Now, let us group html tags into sets. *(You do not have to remember what these tags do to answer the questions; if you want, you can pretend they are just random words.)*

E={html, body, head}
F={{table, tr, td}, aHref, font, h1}
G={{head, title, meta}, {form, textarea, {input, radio}}}
H={h1, h2, h3, center, u}

Answer true or false. If false, explain why – for example, how you can change the equation to make it true.
|H|=5
|F|=6   false, |F|=4, because {table, tr, td} counts as a single element
{body, head} ⊆ E   true
meta ∈ G
td ∈ F
|G|=6
{html} ∈ E
|P(E)|=8   true ($|P(E)|=2^{|E|}$)

Give the elements based on the following set notation:

J={y^2 : y∈ ℕ}
K={x : 3x∈ ℤ and x≥-2}
L={y : |y|<4 and y∈ ℤ }   L={-3,-2,-1,0,1,2,3}
M={z | z∈ ℕ and z^2=4}

A class of 17 students takes a trip to Microsoft headquarters in Redmond, Washington. 10 students want to see the Xbox lab. 9 students want to see the secret teleportation lab. (Each student wants to see at least one of these two labs.) How many students want to go to both labs?

Each student in a CISC 1100 class will receive one or two prizes for the web site they wrote. The two prizes available are a new motorcycle and an old goat. There are 15 students in the class. 7 receive a goat and a motorcycle. 3 receive just a goat. How many students total receive a motorcycle (with or without the goat)?

We are constructing a world in the Alice programming environment and make several statements about this world. (*You do not have to remember anything about Alice programming to answer this question, but hopefully it will be more fun to think about if you do remember.*)
a = Alice is near the barn.
b =The barn door is closed.
c = A cow is coming out of the barn.
d = A penguin dances in front of Alice.

Write each of the following using the propositional variables a, b, c, and d.
Alice is near the barn and a penguin in not dancing in front of Alice.

If a cow is coming out of the barn and Alice is not near the barn, the barn door is not closed.
c∧a'→b'

A cow is coming out of the barn if and only if a penguin dances in front of Alice. Furthermore, Alice is not near the barn and the penguin is not dancing in front of Alice.

Give the truth table of:

a∧b'→b

(b∨a)'

a∨a'

a	a'	a∨a'
T	F	T
F	T	T

b⊕c

b	c	b⊕c
T	T	F
T	F	T
F	T	T
F	F	F

Use truth tables to determine whether the following expression pairs are equivalent

(p∧q)'∨r ≡? (q→r) ∨p'

r∧p⊕q ≡? r↔q∨p

r∧q∧(p∨r') ≡? r∧(p'∨q')'

P	q	r	r'	p∨r'	r∧q	**r∧q∧(p∨r')**	p'	q'	(p'∨q')	(p'∨q')'	**r∧ (p'∨q')'**
T	T	T	F	T	T	**T**	F	F	F	T	**T**
T	T	F	T	T	F	**F**	F	F	F	T	**F**
T	F	T	F	T	F	**F**	F	T	T	F	**F**
T	F	F	T	T	F	**F**	F	T	T	F	**F**
F	T	T	F	F	T	**F**	T	F	T	F	**F**
F	T	F	T	T	F	**F**	T	F	T	F	**F**
F	F	T	F	F	F	**F**	T	T	T	F	**F**
F	F	F	T	T	F	**F**	T	T	T	F	**F**

equivalent

Apply propositional laws to find equivalent expression:

For example a∨a≡a using idempotent law (you don't have to name the law you are using)

$(c \vee d)'$

$[(b \rightarrow a)']' \equiv b \rightarrow a$

$(a \wedge b) \wedge c$

I have written my own version of the Alice programming environment called the "Daniel programming environment." There are five characters/objects in the Daniel world: a professor, a student, a robot, a rabbit, and a duck. I define several "functions". For each, answer:

- Is it a valid function?
- Is it an injection?
- Is it a surjection?
- Is it a bijection?
- Is it invertible?

$height_1$: {professor, student, robot, rabbit, duck} -> {1, 2, 3, 4, 5}

object	professor	student	robot	rabbit	Duck
$height_1$(object)	5	4	1	3	2

$height_2$: {professor, student, robot, rabbit, duck} -> $\mathbb{N}$

Object	professor	student	robot	rabbit	Duck
$height_2$(object)	5	4	1	3	2

Is function; is injection; is not surjection; is not bijection, is not invertible

$height_3$: {professor, student, robot, rabbit, duck} -> $\mathbb{N}$

Object	professor	student	robot	rabbit	Duck
$height_3$(object)	6.4	4.23	.4	2.5	2

$height_4$: {professor, student, robot, rabbit, duck} -> {1, 4, 5}

Object	professor	student	robot	rabbit	Duck
$height_4$(object)	5	4	1	1	4

Is function; is not injection; is surjection; is not bijection; is not invertible

The duck in my Daniel world is always happier than the rabbit. I provide three happiness functions below. Find the inverse for each function. *(If the happiness example does not make sense to you, forget about the animals and just find the inverse.)*

$happy_1$: $\mathbb{R} \to \mathbb{R}$ $happy_1(x)=3x-2$

$happy_2$: $\mathbb{Z} \to \mathbb{Z}$ $happy_2(y)=y+4$

$happy_3$: $\mathbb{R}^{\geq 0} \to \mathbb{R}^{\geq 4}$ $happy_3(y)=(y+2)^2$

$happy_3^{-1}$: $\mathbb{R}^{\geq 4} \to \mathbb{R}^{\geq 0}$ $happy_3^{-1}(z)=\sqrt{z}-2$

If we change the domain and co-domain for $happy_3$ so we now have:
$happy_4$: $\mathbb{Z} \to \mathbb{Z}$ $happy_4(y)=(y+2)^2$
the function is no longer invertible. Why is it no longer invertible? Specifically:

- Is it still an injection?
- Is it still a surjection?
- Is it still a bijection?

I have a new student and I have taught her 5 html tags. I give her a lab assignment in which she must use at least 20 html tags. Presuming she only uses the tags I taught her, can I be guaranteed she will use at least one tag more than once? Explain why or why not using the Pigeonhole principle.

I have gotten a big new office with 12 windows out onto campus. 5 pigeons fly into my office (presumably to ask me about functions). Can I be guaranteed that at least two pigeons fly through the same window to get into my office? Explain why or why not using the Pigeonhole principle.

I cannot be guaranteed at least two pigeons will fly into the same window. The number of windows $|W|$ is greater than the number of pigeons $|P|$ --- $|W|>|P|$. In order to guarantee multiple pigeons through one window, the Pigeon-window principle (er, “pigeonhole principle”) requires $|W|<|P|$.

Compute the following function compositions. You may assume all compositions here are valid. (On the actual exam I may ask you whether a few examples are valid or ill-posed.)

f: $\mathbb{Z} \to \mathbb{N}$ f(x)= $\begin{cases} x \text{ if } x \geq 0 \\ 0 \text{ if } x < 0 \end{cases}$

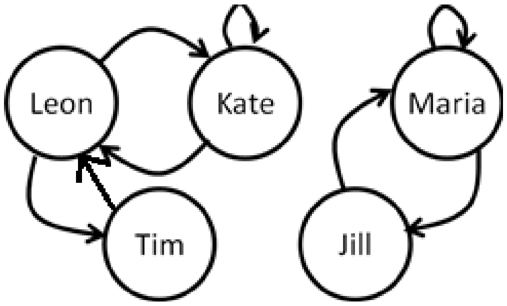
g: $\mathbb{R} \to \mathbb{R}$ g(y)=$2y^2+2$

g°f(-4)
g°f(8)
f°f(12) f(f(12)) = f(12) = **12**
g°g(1) g(g(1)) = $g(2\times1^2+2)=g(4)=2\times4^2+2=2\times16+2$ = **34**

h: $\mathbb{R} \to \mathbb{R}$ h(y)=y-5
k: $\mathbb{R} \to \mathbb{R}$ k(x)=4x
h°k(y)
k°k(y) = k(k(y))=k(4y)=4×(4y)=16y
h°k(-3)

For each of the three relations defined below:
- Draw a graph (circles and arrows) corresponding to the relation
- Say whether the relation is:
  + reflexive, irreflexive, neither
  + symmetric, anti-symmetric, neither
  + transitive, not-transitive

Relation 1, r_1, on the set of people {Leon, Jill, Maria, Tim, Kate}
r_1 = {(Leon, Kate), (Kate, Leon), (Kate, Kate), (Maria, Jill), (Jill, Maria), (Maria, Maria), (Tim, Leon), (Leon, Tim)}



Not reflexive, symmetric, not transitive

Relation 2, r_2, on the set of food {pizza, fries, hotdog, burger, soda}
r_2 = {(soda, soda), (soda, hotdog), (soda, pizza), (burger, fries), (fries, burger), (fries,fries), (pizza, fries), (pizza, burger) }

Relation 3, r_3, on the set of numbers {1,2,3,4,5,6,7,8}

r_3 = {(1, 1), (1,4), (1,8), (3, 3), (4, 4), (4,8), (5, 5), (5,8), (8, 8)}

Write out the set of ordered pairs in the following relations on the integers $\mathbb{Z}$:

(x,y) is in the relation if and only if y > 3x

(x,y) is in the relation if and only if 3x-y=4

(x,y) is in the relation if and only if $\frac{x}{y}$=5

(x,y) is in the relation if and only if x-3=2y

{…, (-3,-3), (-1,-2), (1,-1), (3,0), (5,1), …}

Consider the following relations on the set of all people and say whether the resulting relations are: reflexive, irreflexive, or neither; symmetric, anti-symmetric, or neither; transitive or not

Has as many siblings as

Reflexive, symmetric, transitive

Is shorter than

Has bought food at the same restaurant as

Took the same Spring 2014 classes as

In how many ways can 1 gold, 1 silver, and 1 bronze metal be awarded to 10 contestants?

P(10,3)=10x9x8=**720**

How many ways are there to select 3 class representatives from a class of 25 students?

You can borrow up to 3 DVDs from your friends, who owns 30 DVDs.

- How many ways are there to select 1 DVD?
  C(30,1)=**30**
- How many ways are there to select 2 DVDs?
  C(30,2)=$\frac{30!}{28!2!} = \frac{30\times29}{2\times1}$ =30x29/2=15x29=**435**
- How many ways are there to select 3 DVDs?
  C(30,3)=$\frac{30\times29\times28}{3\times2\times1}$=30x29x28/6=5x29x28=**4060**
- How many ways are there to select 3 or fewer DVDs?
  C(30,3)+C(30,2)+C(30,1)+C(30,0)=4060+435+30+1=**4526**

There are 8 songs you can play at your party. How many ways are there to select a play list containing 5 of these songs (where the order matters)?

You are taking a multiple choice test where each question has 5 possible answers to choose from --- A, B, C, D, or E --- and there are 4 questions in total.

- How many possible ways are there to select your answers?

- How many ways are there to select answers if no letter can be used for more than one question?
- How many ways are there to select answers if the fourth answer must by the same as the first?
  5x5x5x1=**125**
- How many ways are there to select answers if one letter is used for two of the answers and the remaining answers use distinct letters?

You order a computer science book as a gift for your best friend (computer science books make the best gifts). It is available

- in hardcover and in paperback
- new and used
- in first, second, and third edition
- for overnight, 2-day, and 5-day delivery

How many ways can you order the computer science book?

I am forming a Fordham Jeopardy team and need to select 6 members. I have 7 candidates from the Computer Science department, 9 from the Economics department, 10 from the English department, and 12 from the Theater department.
(Give the expression to find the answers, actual number is welcome but not required)

- How many ways can I choose from all the candidates?
- How many ways can I choose a team where all members are from the same department?
- How many ways can I choose a team with 2 computer science students, 2 economics students, and 2 theatre students?
  C(7,2)+C(9,2)+C(12,2)=$\frac{7!}{5!2!}+\frac{9!}{7!2!}+\frac{12!}{10!2!}$ = 7x6/2+9x8/2+12x11/2 =
  7x3+9x4+6x11=21+36+66=**123**
- How many ways can I choose a team with no computer science majors?

Solve for:

P(3,1)

P(4,2)=4x3=**12**

P(5,2)

P(m,0)

C(5,3)=$\frac{5!}{3!2!}=\frac{5\times4\times3}{3\times2\times1}$ =5x4x3/6=60/6=**10**

C(4,1)=4/1!=**4**

C(4,4)

You are given an exam with every question having either the answer "true" or "false." There are 10 questions on the exam and you guess randomly for each question. Assuming each answer is equally likely to be true or false:

- What is the probability you get exactly 5 questions correct?
- What is the probability you get fewer than 8 questions correct?
  1-Prob(get 8 or more correct)
  Prob(get 8 or more correct)=Prob(exactly 8 correct)+Prob(exactly 9 correct)
  +Prob(exactly 10 correct)
  Prob(exactly 8 correct)=C(10,8)/2^{10}=**45/2^{10}** (arrangements of CCCCCCCCII, where C is "correct" and I is "incorrect")
  Prob(exactly 9 correct)=C(10,9)/2^{10}=**10/2^{10}**
  Prob(exactly 10 correct)=C(10,10)=**1/2^{10}**
  $1-\frac{(45+10+1)}{2^{10}}=1-\frac{56}{2^{10}}=\mathbf{\frac{968}{1024}\approx .945}$

You have a jar of jelly beans in front of you containing 20 cherry, 10 orange, 15 lemon, 15 watermelon, and 10 grape flavor. You take 5 beans without looking.
(Give the expression to find the answers, actual number/fraction is welcome but not required):

- What is the probability you pick one of each flavor?
- What is the probability none of the picked beans are cherry?

We go to the casino and play a dice-rolling game. Each die is 6 sided and 3 die are rolled.

- What is the probability all 3 die roll higher than 4?
- What is the probability at least one 3 is rolled?
  ~~Prob(exactly one 3)+Prob(exactly two 3s)+Prob(exactly three 3s)= $\frac{3+3+1}{216}=\frac{7}{216}$~~
  1-Prob(no 3's are rolled)=$1-\frac{5\times5\times5}{6\times6\times6}\approx 1-0.58=\mathbf{0.42}$
- What is the probability all 3 rolls are different?
- What is the probability the first 2 dice rolled add up to 2 or 5?
- What is the probability the first roll is even or greater than 4?
  Prob(even)+Prob(greater than 4)-Prob(even and greater than 4)=
  $$\frac{3}{6}+\frac{2}{6}-\frac{1}{6}=\frac{4}{6}=\mathbf{\frac{2}{3}}$$
- What is the probability the second roll is a multiple of 2 or of 3?

I flip a biased coin where it will land on heads with 2/3 probability.

- What is the probability of getting 2 tails in 2 flips?
- What is the probability of getting all heads in 4 flips?
  Prob(first head)xProb(second head)xProb(third head)xProb(fourth head)=
  $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \mathbf{\frac{16}{81}}$
- What is the probability of getting fewer than 2 tails in 4 flips?