

Presume we would like to model the axon voltage/spiking of a neuron using the differential equation and voltage reset rule we used in class.

Let us assume the variable values for the model neuron are

$$E_L = -60\text{mV} \quad v_{\text{reset}} = -60\text{mV} \quad \tau = 0.5 \quad RI(t) = 80\text{ mV (for all time)} \quad v(0\text{ms}) = -60\text{mV} \\ v_{\text{thresh}} = 40\text{ mV}$$

Using a time step  $\Delta t = 0.001$  second (1 ms), compute  $v(3\text{ms})$

Will the neuron ever spike? If not, why not?

Now let us change  $RI$ . Specifically, let us assume the variable values for the model neuron are

$$E_L = -60\text{mV} \quad v_{\text{reset}} = -60\text{mV} \quad \tau = 0.1 \quad RI(t) = 100\text{ mV (for all time)} \quad v(0\text{ms}) = -60\text{mV} \\ v_{\text{thresh}} = 35\text{ mV}$$

Using a time step  $\Delta t = 0.001$  second (1 ms), compute  $v(3\text{ms})$

$$t = 0.001\text{ s: } \Delta v = \frac{1}{\tau} [-(v(0) - E_L) + RI(0)] = \frac{1}{0.1} [ -(-0.06 - (-0.06)) + 0.1 ] 0.001$$

$$= 10 \times [0 + 0.1] \times 0.001 = 0.001\text{ V}$$

$$v(1) = v(0) + \Delta v = -0.06 + 0.001 = -0.059\text{ V (-59 mV)}$$

$$t = 0.002\text{ s: } \Delta v = \frac{1}{0.1} [ -(-0.059 - (-0.06)) + 0.1 ] 0.001 = 10 \times [-0.001 + 0.1] \times 0.001$$

$$= 10 \times [0.099] \times 0.001 \approx 0.00099\text{ V}$$

$$v(2) = v(1) + \Delta v = -0.059 + 0.00099 = -0.05801\text{ V (-58.0 mV)}$$

$$t = 0.003\text{ s: } \Delta v = \frac{1}{0.1} [ -(-0.005801 - (-0.06)) + 0.1 ] 0.001 =$$

$$10 \times [-0.00199 + 0.1] \times 0.001 = 10 \times [0.09801] \times 0.001 \approx 0.00098\text{ V}$$

$$v(3) = v(2) + \Delta v = -0.058 + 0.00098 = -0.05702\text{ V (-57 mV)}$$

Will the neuron ever spike? If not, why not?

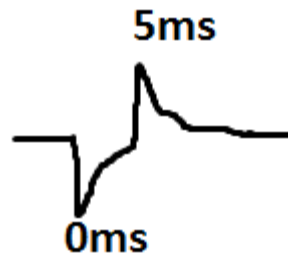
**The neuron will spike.  $E_L + RI = -60 + 100 = 40\text{mV}$ , which is greater than  $35\text{mV}$ , so the neuron will reach above the threshold voltage and will reset.**

Can changing the variable  $\tau$  (with the requirement that it remains a positive number) prevent the neuron from firing? Why or why not?

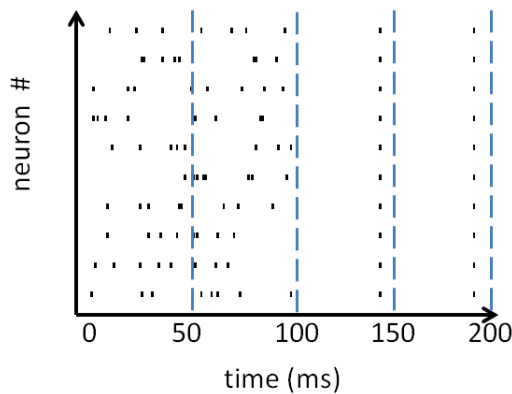
**$\tau$  only changes the speed at which the voltage changes. It does not change the maximum voltage the neuron can rise to. Therefore, changing  $\tau$  would not prevent the neuron from firing.**

Draw the input over time,  $R_I(t)$ , resulting from the firing of an inhibitory pre-synaptic neuron at time  $t=0\text{ms}$  and a second inhibitory pre-synaptic neuron at time  $t=5\text{ms}$ . (For the purpose of this question, all that matters is general shape.)

Draw the input over time,  $R_I(t)$ , resulting from the firing of an inhibitory pre-synaptic neuron at time  $t=0\text{ms}$  and an excitatory pre-synaptic neuron at time  $t=5\text{ms}$ . (For the purpose of the is question, all that matters is general shape.)



Consider the following spiking pattern, produced by a set of 10 neurons.



What is the average spike rate from 0 to 100 ms?  
 Count number of spikes for each neuron between 0 and 100 ms. Neuron 1: 7, Neuron 2: 6, Neuron 3: 7, Neuron 4: 7, Neuron 5: 8, Neuron 6: 6, Neuron 7: 7, Neuron 8: 7, Neuron 9: 8, Neuron 10: 8:

$$\frac{7+6+7+7+8+6+7+7+8+8}{10} \approx 7$$

$$\underline{7/.1 = 70 \text{ spikes/sec}}$$

What is the average spike rate from 100 to 200 ms?

$$\frac{2+2+2+2+2+2+2+2+2+2}{10} = 2$$

$$\underline{2/.1 = 20 \text{ spikes/sec}}$$

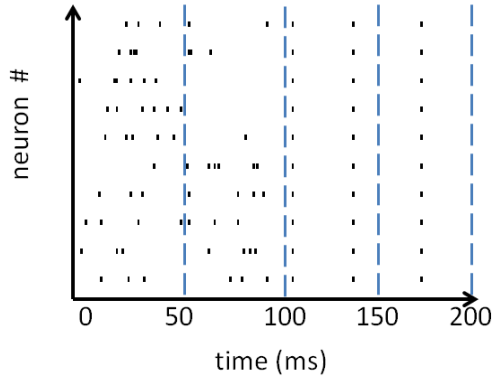
Which of the 50ms windows indicates a strong rate-based response? (You can choose multiple windows. Choose windows between the blue dashed lines, e.g., 0-50ms.)

**0-50ms (possibly 50-100ms)**

Which of the 50 ms windows indicates a strong timing-based response, where significant inputs cause global synchrony within the neural population? (You can choose multiple windows. Choose windows between the blue dashed lines, e.g., 0-50ms.)

**100-150 ms, 150-200ms**

Consider the following spiking pattern, produced by a set of 10 neurons.



What is the average spike rate from 0 to 100 ms?

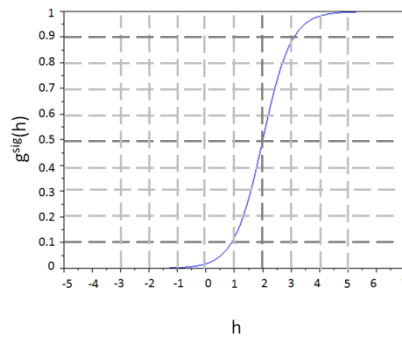
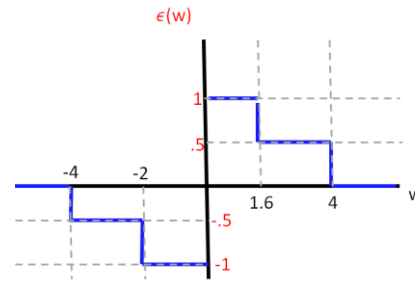
What is the average spike rate from 100 to 200 ms?

Which of the 50ms windows indicates a strong rate-based response? (You can choose multiple windows. Choose windows between the blue dashed lines, e.g., 0-50ms.)

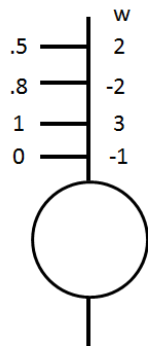
Which of the 50 ms windows indicates a strong timing-based response, where significant inputs cause global synchrony within the neural population? (You can choose multiple windows. Choose windows between the blue dashed lines, e.g., 0-50ms.)

Compute learned weights based on initial weights and input rates using:

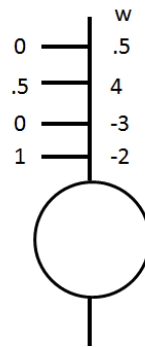
- (a) Hebb learning with  $\epsilon(w)$  specified to the right
- (b) Willshaw learning
- (c) Hebb learning with  $\epsilon=1$  and normalization to a combined weight of 1



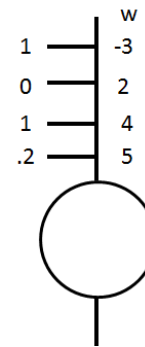
The output of each unit is computed using the weighted sum and the sigma non-linearity  $g^{\text{sig}}$ .



Example 1



Example 2



Example 3

$$r_i = g^{\text{sig}}(1 \times -3 + 0 \times 2 + 1 \times 4 + .2 \times 5) = g^{\text{sig}}(-3 + 0 + 4 + 1) = g^{\text{sig}}(2) \approx 0.5$$

Hebb:  $\Delta w_1 = \epsilon(w_1) r_i r_j = -0.5 \times 0.5 \times 1 = -0.25 \rightarrow w_1 = -3.25$

$$\Delta w_2 = \epsilon(w_2) r_i r_j = 0.5 \times 0.5 \times 0 = 0 \rightarrow w_2 = 2$$

$$\Delta w_3 = \epsilon(w_3) r_i r_j = 0.5 \times 0.5 \times 1 = +0.25 \rightarrow w_3 = 4.25$$

$$\Delta w_4 = \epsilon(w_4) r_i r_j = 0 \times 0.5 \times 0.2 = 0 \rightarrow w_4 = 5$$

Willshaw:  $\Delta w_1 = r_i r_j - r_i w_1 = 0.5 \times 1 - 0.5 \times -3 = 0.5 + 1.5 = +2 \rightarrow w_1 = -1$

$$\Delta w_2 = r_i r_j - r_i w_2 = 0.5 \times 0 - 0.5 \times 2 = 0 - 1 = -1 \rightarrow w_2 = 1$$

$$\Delta w_3 = r_i r_j - r_i w_3 = 0.5 \times 1 - 0.5 \times 4 = 0.5 - 2 = -1.5 \rightarrow w_3 = 2.5$$

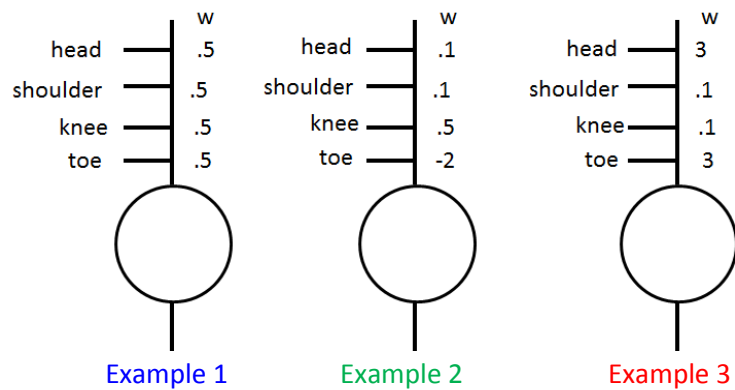
$$\Delta w_4 = r_i r_j - r_i w_4 = 0.5 \times 0.2 - 0.5 \times 5 = 0.1 - 2.5 = -2.4 \rightarrow w_4 = 2.6$$

Hebb+Normalize:  $\Delta w_1 = \epsilon r_i r_j = 1 \times 0.5 \times 1 = 0.5 \rightarrow w_1 = -2.5$   
 $\Delta w_2 = \epsilon r_i r_j = 1 \times 0.5 \times 0 = 0 \rightarrow w_2 = 2$   
 $\Delta w_3 = \epsilon r_i r_j = 1 \times 0.5 \times 1 = 0.5 \rightarrow w_3 = 4.5$   
 $\Delta w_4 = \epsilon r_i r_j = 1 \times 0.5 \times 0.2 = 0.1 \rightarrow w_4 = 5.1$

Normalizing:  $-2.5 + 2 + 4.5 + 5.1 = 9.1$

$w_1 = -2.5/9.1 \approx -0.3$      $w_2 = 2/9.1 = 0.2$      $w_3 = 4.5/9.1 = 0.5$      $w_4 = -5.1/9.1 \approx 0.6$

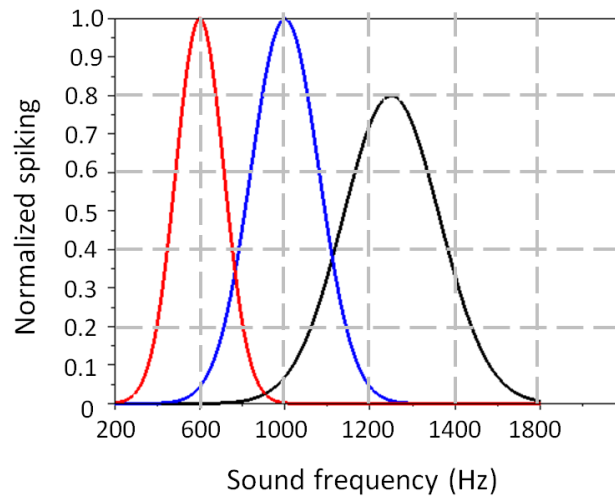
Below, we consider three example neurons. Each one computes a weighted sum  $h$  from four inputs, and output  $r_{out} = \begin{cases} 0 & \text{if } h < 1.5 \\ 1 & \text{if } h \geq 1.5 \end{cases}$ . Each input – head, shoulder, knee, and toe – has the value 1 when the named body part is seen and has the value 0 when the named body part is **not** seen. State whether each neuron below performs generalization, performs prototype recognition, or does neither of the two previously mentioned tasks.



Example 1 performs ~~prototype recognition~~ **generalization**.

Sound curves,

Let us consider neurons in the cochlea. Recording from three of these cells in a cat, we find each cell fires at a normalized rate indicated by the curve below. The blue curve is for neuron A, the red curve for neuron B, and the black curve for neuron C.



What sound frequency could the cat be hearing given the three neurons fire at the following rates. (You can round to the nearest 100 Hz.)

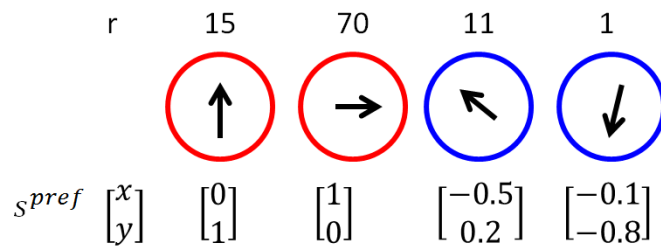
$\hat{r}_A=0.7, \hat{r}_B=0, \hat{r}_C=0.2$

$\hat{r}_B=0.4, \hat{r}_A$  and  $\hat{r}_C$  unmeasured

$\hat{r}_A=0, \hat{r}_B=0.9, \hat{r}_C=0$

**500 Hz**

We record from several neurons in the motor cortex representing the desired direction of a monkey's right leg. Each neuron represents motion in a particular direction at a particular speed, as specified by the vector  $\begin{bmatrix} x \\ y \end{bmatrix}$  where x indicates left (-1) or right (+1) and y indicates backwards (-1) or forwards (+1).  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$  would correspond to the leg moving forward at a speed of 1 and, at the same time, to the right at a speed of 1. **For blue-colored neurons, the minimum firing rate is 1 Hz and the maximum is 20 Hz. For red-colored neurons, the minimum firing rate is 10 Hz and the maximum is 80 Hz.** Using population coding (computing  $\hat{s}_{pop}$ ), what is the represented movement of the leg (direction and speed) given by the following rates r?



$$\hat{r}_1 = \frac{15-10}{80-10} = \frac{5}{70} \approx 0.07$$

$$\hat{r}_2 = \frac{70-10}{80-10} = \frac{60}{70} \approx 0.86$$

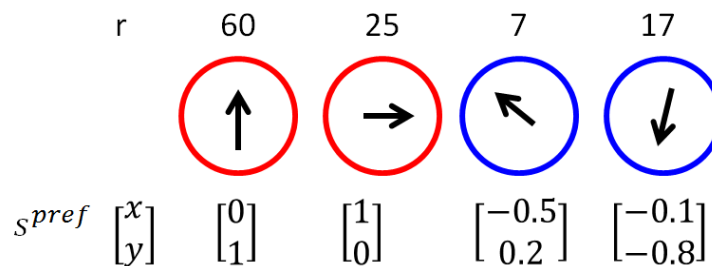
$$\hat{r}_3 = \frac{11-1}{20-1} = \frac{10}{19} = 0.53$$

$$\hat{r}_4 = \frac{1-1}{20-1} = \frac{0}{19} = 0$$



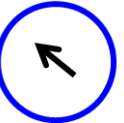

$$(0.07+0.86+0.53+0)=1.46$$

$$\frac{0.07}{1.46} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \frac{0.86}{1.46} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{0.53}{1.46} \begin{bmatrix} -0.5 \\ 0.2 \end{bmatrix} + \frac{0}{1.46} \begin{bmatrix} -0.1 \\ -0.8 \end{bmatrix} \approx \begin{bmatrix} 0 \\ 0.05 \end{bmatrix} + \begin{bmatrix} 0.59 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.18 \\ 0.07 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.41 \\ 0.12 \end{bmatrix} \approx \begin{bmatrix} 0.4 \\ 0.1 \end{bmatrix}$$

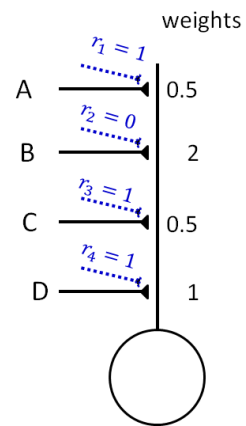
What is the represented movement of the leg (direction and speed) given by these following rates r?



What is the represented movement of the leg (direction and speed) given by these following rates r?

$r$	45	15	15	3
				
$s^{pref} \begin{bmatrix} x \\ y \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} -0.5 \\ 0.2 \end{bmatrix}$	$\begin{bmatrix} -0.1 \\ -0.8 \end{bmatrix}$

Consider a model neuron whose output is determined solely by the weighted sum of its inputs. However, each input is modulated by a further attention weight, indicated by the dashed blue lines in the diagram at right. Given the rates for the four inputs A, B, C, and D, compute the output.



Input 1:  $r_A=0$   $r_B=1$   $r_C=0.5$   $r_D=0$   
 Input 2:  $r_A=0.5$   $r_B=0.3$   $r_C=0.1$   $r_D=1$

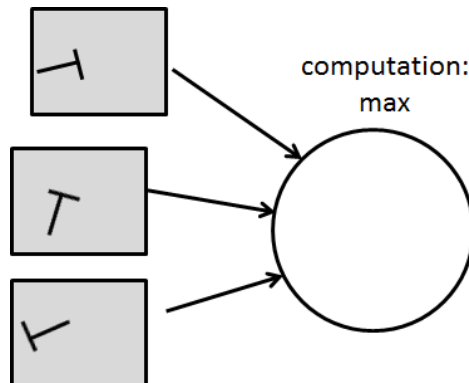
Which input (or inputs) are ignored based on these attention weights?

**B**

Alternative HMAX patterns

This neuron has invariance to which of the following transformations?

- (a) Angle (rotation)
- (b) Size (grow/shrink)
- (c) Location (movement along x or y axis)

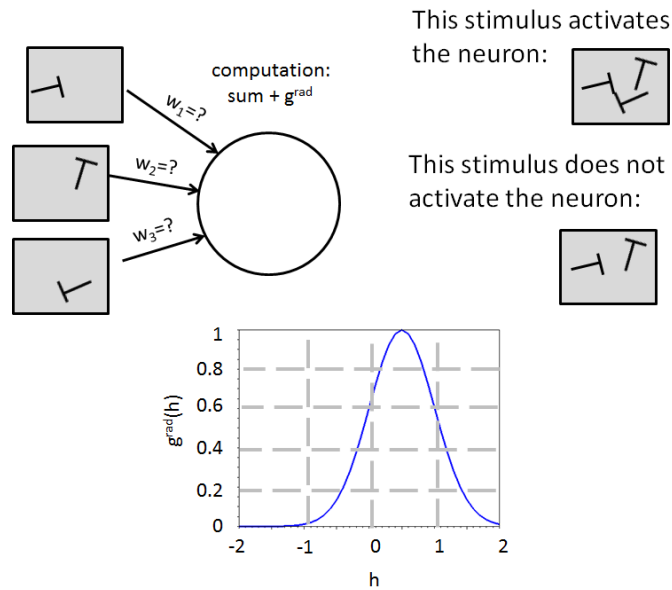


**(a)**





Find the appropriate weights that will allow the following neuron to activate (fire higher than 0.85) for the first stimulus but not for the second stimulus.



Compute the following matrix products:

$$\begin{bmatrix} 4 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 4 & -1 \\ 2 & 3 \end{bmatrix} =$$

$$\begin{bmatrix} 5 & -5 \\ 2 & 0 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \times 3 - 5 \times 2 \\ 2 \times 3 + 0 \times 2 \\ -1 \times 3 + 4 \times 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 3 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \\ 8 \end{bmatrix} =$$

We perform an experiment with 10 freshman subjects and 10 senior subjects. Each subject sits in front of a computer monitor. Every 10 seconds, the subject is shown a math problem and 4 potential answers, such as:

What is  $5+4$ ?

- a) 12
- b) 2
- c) 9
- d) 100

The subject must select the correct answer as quickly as possible for each question, pressing button a, b, c, or d on a keypad. The subject is shown 100 questions. The subject's response time for each question is recorded.

After performing this experiment, the data is stored in  $20 \times 100$  matrix called `ResponseTimes`, where the first 10 rows are for the 10 freshmen and the second 10 rows (rows 11 through 20) are for the seniors. Response times are recorded in the order of the questions asked – the 30<sup>th</sup> question asked is recorded as the 30<sup>th</sup> response time in each row. Provide Matlab command(s) to answer the following questions:

What is the average response time of the freshmen?

How much faster does each senior answer the last question they are asked than the first question they are asked?

Is there a greater correlation between two seniors' response times across all questions than between freshmen's response times across all questions? (You can pick any two seniors and any two juniors to answer this question.)

**Check correlation of seniors:** `corr(ResponseTimes(11, :), ResponseTimes(12, :))`

**Check correlation of freshmen:** `corr(ResponseTimes(1, :), ResponseTimes(2, :))`

For each memory, say if it is declarative or non-declarative. If relevant, also say if it is long-term or short-term.

Remembering how to play the guitar

Remembering what you ate for a mid-day snack ten minutes ago.

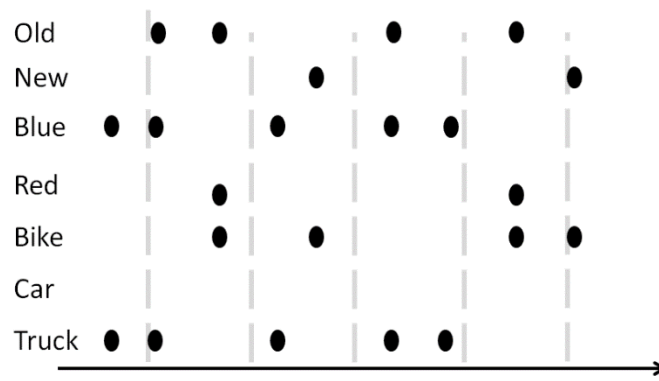
Remembering Harry Potter is a wizard.

**Declarative, long-term**

Remembering that two vertical lines surrounding a horizontal line creates the letter H.

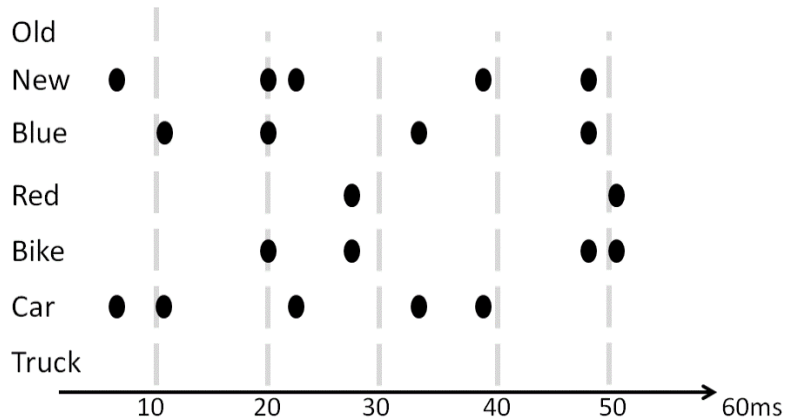
Reflecting the binding hypothesis, list the set of objects (e.g., “shiny red car,” “yellow submarine”) present in the scene producing the following spiking patterns. Each row reflects the spiking of a neuron encoding the feature named at the beginning of the row. In this assignment, spikes are considered to be synchronous if they occur within 1 ms of one another.

**a:**



**Blue truck, Old blue truck, Old red bike, New bike, New bike**

**b:**



If we redefine synchrony as spikes occurring within 3 ms of one another, which of the two above firing patterns may suffer from spurious synchronization? List two pairs of object most likely to be confused with each other.

A monkey wishes to move his arm from next to his body – a distance of 0 m from his body – to an outstretched position – a distance of 3 m from his body. He will use the strategy of moving a distance  $\Delta\text{location}$  at each time step, where

$$\Delta\text{location} = (\text{desiredPosition} - \text{currentPosition}) \times \text{multiplier}$$

He also will use the basic feedback system to control his motion.

Compute the arm locations at the first 5 time steps after motion has begun, presuming *multiplier*=0.2

t	1	2	3	4	5	6
currentPosition	0					

Compute the arm locations at the first 5 time steps after motion has begun, presuming *multiplier*=0.4 and the arm is pushed backwards by 0.5 m at time t=2.

t	1	2	3	4	5	6
currentPosition	0					

Compute the arm locations at the first 5 time steps after motion has begun, presuming *multiplier*=0.6 and there is a 3 time-step delay in afferent feedback.

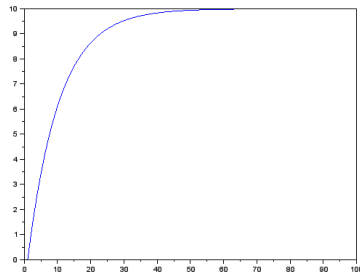
t	1	2	3	4	5	6
currentPosition	0	1.8	3.6	<u>5.4</u>	<u>7.2</u>	<u>7.9</u>
sensedPosition	0	0	0	<u>0</u>	<u>1.8</u>	



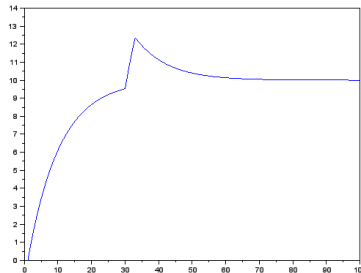
Below we plot the motion of an arm stretching to a desired distance, using the basic feedback system.

Which motion reflects a delay in sensory feedback? c

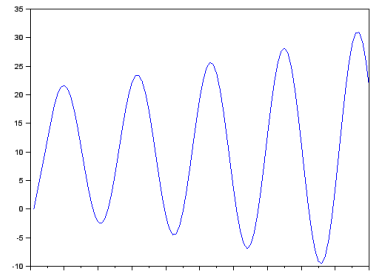
Which motion reflects someone jolting the arm in mid-motion?



(a)

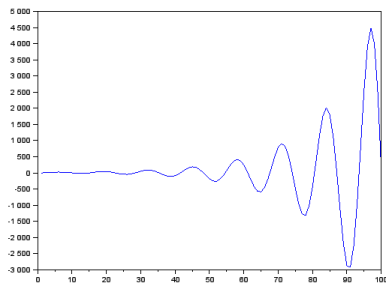


(b)

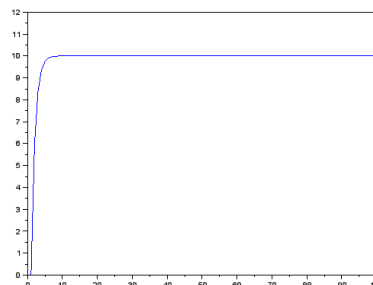


(c)

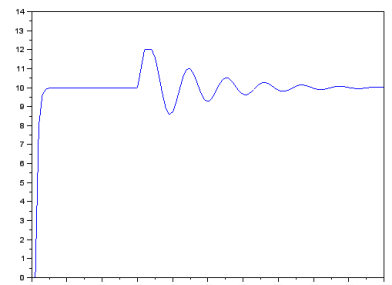
Below we plot several potential arm motions for an arm stretching to a desired distance, with delayed sensory feedback. Which motion (or motions) could reflect use of a forward model, rather than just a basic feedback system?



(a)

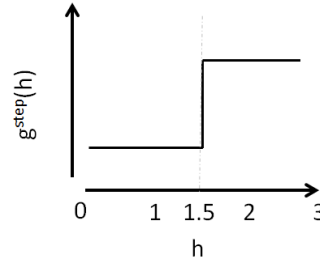
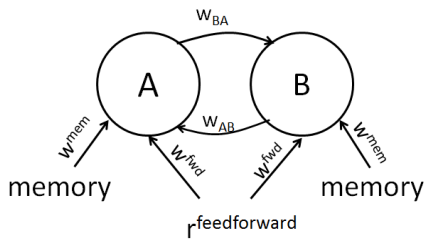


(b)



(c)

We are given two model neurons receiving feedforward input and memory input, as well as providing lateral input to each other. Each neuron's firing output  $r^{out}$  is determined by computing the weighted sum  $h = \sum_j w_j r_j$  and using a step function activation function  $g^{step}(h) = \begin{cases} 0 & \text{if } h < 1.5 \\ 1 & \text{if } h \geq 1.5 \end{cases}$ .



We assume the weights on the memory and  $r^{\text{feedforward}}$  inputs to both neurons are 1:

$$w^{\text{mem}}=0.5, w^{\text{fwd}}=2$$

For this homework, we will explore the effects of different lateral weights.

i) Given  $w_{AB}=0$  and  $w_{BA}=2$ , and given the following memory and feedforward inputs, compute the outputs for neurons A and B over time:

Time	1	2	3	4	5	6
$r^{\text{feedforward}}$	1	1	1	0	0	0
memory	0	0	0	1	1	1
$r_A^{\text{out}}$	0					
$r_B^{\text{out}}$	0					

ii) Given  $w_{AB}=2$  and  $w_{BA}=-2$ , and given the following memory and feedforward inputs, compute the outputs for neurons A and B over time:

Time	1	2	3	4	5
$r^{\text{feedforward}}$	1	1	0	0	0
memory	1	1	1	1	0
$r_A^{\text{out}}$	0	<b>1</b>	<b>1</b>	<b>0</b>	<b>0</b>
$r_B^{\text{out}}$	0	<b>1</b>	<b>0</b>	<b>0</b>	<b>0</b>



iii) Given  $w_{AB}=4$  and  $w_{BA}=2$ , and given the following memory and feedforward inputs, compute the outputs for neurons A and B over time:

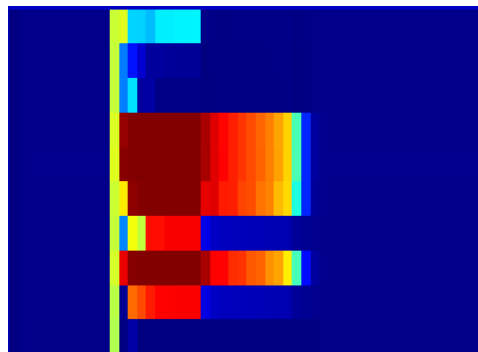
Time	1	2	3	4	5	6
$r^{\text{feedforward}}$	0	1	0	0	0	0
memory	0	1	1	1	0	0
$r_A^{\text{out}}$	0					
$r_B^{\text{out}}$	0					

Consider the firing patterns over time for different sets of 10 neurons shown here.

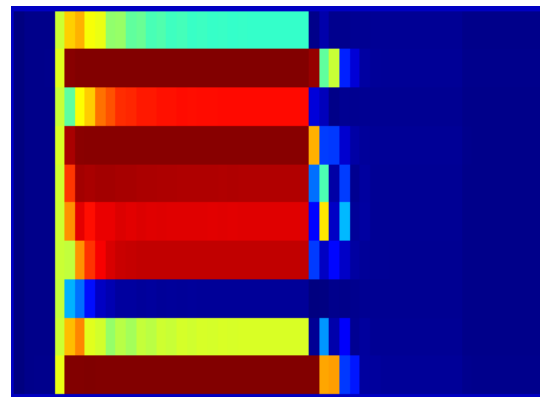
a) Indicate whether each is a memory activity, a decaying activity, or a growing activity.

b) Indicate at **what time** point **the feedforward and memory input begin** (they will always begin simultaneously in these examples), at **what time** point **the feedforward input ends** (switches back from 1 to 0), and at **what time** point **the memory inputs ends** (switches back from 1 to 0). If this is not possible for a given activity, state it is not possible to tell.

i:



ii:

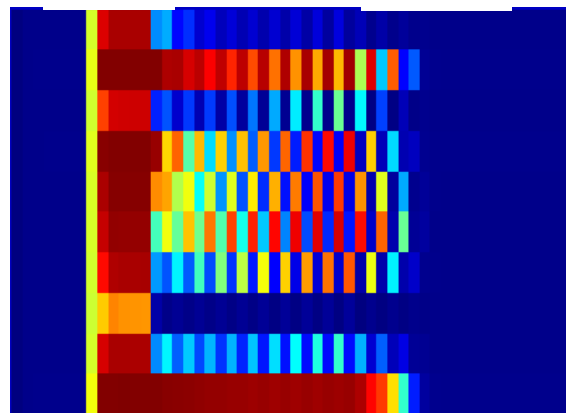


Decay activity

↑  
Start  
input

↑  
End  
feedfwd

iii:



According to our model of the hippocampus, what is the effect of removing the connection from the dentate gyrus to CA3?

According to our model of the hippocampus, what is the effect of reducing the number of lateral/recurrent connections within CA3?

**Decrease association between memories, decrease ability to recall one memory triggered by associated other memories.**