

Homework Assignment #6 (Counting)

Note: Please explain your answers to all questions, e.g., if you use multiplication rule, explain why the rule applies; if you use combination formula, explain why the problem is counting combination. **Such explanations take up half of the points.** Here are some guidelines for how to approach the problem:

- Is this a permutation problem?
- If not, is it a combination problem?
- If not, can we solve it using multiplication rule, by breaking it down to multiple step decision/process, with each step has a fixed number of outcomes?
- Remember that, we might need to analyze the cases/outcomes we are counting into multiple possible situations, count each situation separately and add them together (using inclusion-exclusion principle to get rid of intersection if needed).

1 Calculate the following:

(a) $P(8, 3)$

(b) $C(7, 3)$

2 You have 10 books in the shelf, 4 are textbooks and 6 are novels. Answer the following questions about different ways to arrange these books.

(a) How many different ways are there to arrange all the 10 books?

(b) If you are to put the textbooks together and the novels together, how many different ways are there to arrange the books?

3 In the poker game, a Poker hand of five cards is dealt to the player. The order of the five cards do not matter. Answer the following questions, assuming that the five cards are taken randomly from a deck of 52 cards (with 13 cards from each of the four suits, club, diamond, heart and spade).

a. How many different Poker hands are possible?

b. How many different Poker hands that are **three of a kind**, i.e., three of the cards are of the same rank, and the other two cards are any cards of different rank. E.g., get three 5's (of different suits), one Ace and one 9.

c. How many different Poker hands are **Flush**, i.e., all five cards in the hand are of the same suit.

4 When you toss a fair coin for 8 times and record the result as a sequence of H and T (H for head, T for tail) of the face of the coin that is facing up after each toss. Answer the following questions:

(a) How many different outcomes are possible? For example, HHHHHHHH, HTHTTTTT are examples of two outcomes.

- (b) How many different outcomes have 4 heads and 4 tails?

- (c) How many different outcomes have more heads than tails (e.g., 6 heads and 2 tails, or 5 heads and 3 tails, ...)?

- 5 Of a company's personnel, 7 works in design, 14 in manufacturing, 4 in testing, 5 in sales, 2 in accounting, and 3 in marketing. A committee of 6 people is to be formed to meet with upper management.
 - (a) How many different committees can be formed?

 - (b) In how many ways can the committee be formed if there is to be one member from each department?

 - (c) In how many ways can the committee be formed if there must be exactly two members from manufacturing?

 - (d) In how many ways can the committee be formed if the accounting department is not to be represented and marketing is to have exactly one representative?

