CISC 1100— Structure of Computer Science Fall, 2017

Homework Assignment #4

- **1** Answer the following questions:
 - (a) Let D = 1, 2, 3, Define $F : D \to \mathbf{N}$ by the rule F(n) = 2 3n.
 - i. Is F a function? Why or why not? (Hint: draw arrow diagram for the mapping first)

ii. If we modify the codomain of F to be \mathbf{Z} (the set of all integers), is F a function? If f is a function, is it an injective (i.e., one-to-one) function? Is it a surjective (onto) function? Briefly explain why.

(b) Define $G : \mathbf{R} \to \mathbf{R}$ by the rule G(x) = 2 - 3x for all real numbers x. Is G onto? Explain your answer.

2 Use the space below to draw the so-called arrow diagrams for three functions that are 1) injective but not surjective, 2) not injective but surjective, 3) injective and surjective. You can choose the domain and codomain to be sets containing several elements of your choice.

3 If there are 400 freshman students in Fordham University, then there are at least two students who share the same birthday (i.e, they were born in the same day of the same month, even though not necessarily in the same year). Use Pigeonhole Theorem to explain, and describe what's the domain, codomain and function in applying Pigeonhole Theorem in this example.

- 4 True or False. Explain briefly (you can use diagram to show, or one or two short sentences).
 - (a) For a function $f : X \to Y$, where both domain X and codomain Y are finite sets, if f is an one-to-one function, then $|X| \leq |Y|$.

(b) For a function $f: X \to Y$, where both domain X and codomain Y are finite sets, if f is an onto function, then $|X| \ge |Y|$.

- **5** Let $f : \mathcal{R} \to \mathcal{R}$, and f(x) = 2x + 1 (i.e., function f maps a real number x to a real number given by 2x + 1;), and let $g : \mathcal{R} \to \mathcal{R}$, and $g(x) = x^2$ (i.e., function g maps any real number to its square).
 - (a) Calculate $(g \circ f)(1)$, $(g \circ f)(0.2)$.

(b) What is function $(f \circ g)(x)$ (i.e., write it's formula)? Calculate $(f \circ g)(1)$, $(f \circ g)(0.1)$.

 ${\bf 6}\,$ Find the inverse for each of the following functions:

(a) $f: \mathcal{Z} \to \mathcal{Z}$ defined by f(x) = x + 1.

(b) $g: \mathcal{R} \to \mathcal{R}$ defined by g(x) = 2x + 7.