CISC 2100/2110 — Discrete Structure II
Fall, 2015

Homework Assignment #4

These questions are from the following textbook, Discrete Mathematics with Applications, by Susanna S. Epp.

1 Are the following statements true or false. Prove the statement from the definition if it is true, and give a counterexample if it is false.

(a) For all integers a, b, and c, if $a|bc$ then $a|b$ or $a|c$.

(b) For all integers a and n, if $a|n^2$ and $a \leq n$ then $a|n$.

2 Use the unique factorization theorem to write integer 5733 in standard factored form.

3 If n is an integer and $n > 1$, then $n!$ is the product of n and every other positive integers that is less than n. For example, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$. Without computing the value of $(20!)^2$, determine how many zeros are at the end of this number when it is written in decimal form. Justify your answer. (Hint: $10 = 2 \cdot 5$).

4 Suppose c is an integer. If $c \bmod 15 = 3$, what is $10c \bmod 15$?

5 Prove that the square of any integer has the form of $4k$ or $4k + 1$ for some integer k.

6 If m, n, and d are integers, $d > 0$, and $m \text{ mod } d = n \text{ mod } d$, does it necessarily follow that $m = n$? Does it necessarily follow that $m - n$ is divisible by d? Prove your answer.

7 Is the following statement true or false? Prove your answer. For all odd integers n, $\lceil n/2 \rceil = (n + 1)/2$. (Note, please only use the definition of odd integer, and the ceiling in your proof.)

8 For any integer m and any real number x, if x is not an integer, then $\lfloor x \rfloor + \lfloor m - x \rfloor = m - 1$. (Note: please use only the definition of floor in your proof. Hint: You can start by let $k = \lfloor x \rfloor$, and then use the definition of floor to write an inequation about x.

9 Prove the following statement by **contradiction**: If a and b are rational numbers, $b \neq 0$, and r is irrational number, then $a + br$ is irrational.

10 Prove the following statement by **contradiction**: For all integers a, if $a \bmod 6 = 3$, then $a \bmod 3 \neq 2$.

11 Prove the following statement by **contraposition**: If a sum of two real numbers is less than 50, then at least one of the numbers is less than 25.