CISC 2100/2110 — Discrete Structure II
Fall, 2016

Homework Assignment #4

These questions are from the following textbook, Discrete Mathematics with Applications, by Susanna S. Epp.

Note that please follow the template as provided in 3 (a) for all the problems!. Otherwise, up to one third of the points will be taken off. (This is analogous to the requirement about programming styles in CS1 and CS2).

1 Use the formula for the sum of the first n integers and/or the formula for the sum of a geometric sequence to evaluate the following sums:

(a) $7 + 8 + 9 + 10 + ... + 600$

(b) $1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ... + \frac{1}{2^n}$, where n is a postive integer

(c) $1 - 2 + 2^2 - 2^3 + ... + (-1)^n 2^n$, where n is a postive integer

2 Getting familar with the method of proof by mathematical induction. Consider the following statement:

$$2^n < (n+1)!,$$

for all positive integer $n \geq 2$.

(a) What is $P(n)$, i.e., what is the property defined on integer n that is to be proved here?

(b) What is $P(2)$? Prove that $P(2)$ is true (as the basis step).

(c) Write $P(k)$. Write $P(k+1)$.

(d) Write down what you need to prove as the inductive step of the proof.

3 [You can choose to work on any two of the three] Prove the followin statements by mathematical induction.

(a) $n^3 - 7n + 3$ is divisible by 3, for each integer $n \geq 0$.

Proof. (Proof by mathematical induction)

`1. (basis step, show that the statement is true for n=0).`

`2. (inductive step) We now show that:`

for any integer k ______, if ________ , then _______.
To prove this, let k be any integer, $k > 0$, and suppose _______, (we will show ________)

From the above steps 1 and 2, we have proved the given statement by the principle of mathematical induction. □

(b) $1+nx \leq (1+x)^n$, for all real numbers $x \geq 1$ and integers $n \geq 2$. (Hint: you should/can only apply mathematical induction on n. You should keep x a generic and arbitrary variable throughout your proof, using the knowledge that $x \geq 1$ (this way, the proof works for any real number $x \geq 1$.).

(c) $5^n + 9 < 6^n$, for all integers $n \geq 2$.

4 Evaluate the sum $\sum_{k=1}^{n} \frac{k}{(k+1)!}$ for $n = 1, 2, 3, 4$ and 5. Make a conjecture about a formula for this sum for general n, and prove your conjecture by mathematical induction.

5 [Strong mathematical induction] Suppose that $g_1, g_2, g_3, ...$ is a sequence defined as follows:

$$g_1 = 3, g_2 = 5, g_k = 3g_{k-1} - 2g_{k-2} \text{ for all integers } k \geq 3$$

Prove that $g_n = 2^n + 1$ for all integers $n \geq 1$.

6 [Extra Credits] Use strong mathematical induction to prove the existence part of the unique factorization of integers (Theorem 4.3.5); Every integer greater than 1 is either a prime number or a product of prime numbers.