Theory of Computation
Practice Midterm

Name: _____

Directions:

Answer the questions as well as you can. Partial credit will be given, so show your work where appropriate. Try to be precise in your answers in order to maximize your points. Also make sure that your answers to pumping lemma questions are sufficiently clear so that I can tell that your reasoning is correct. Good luck.

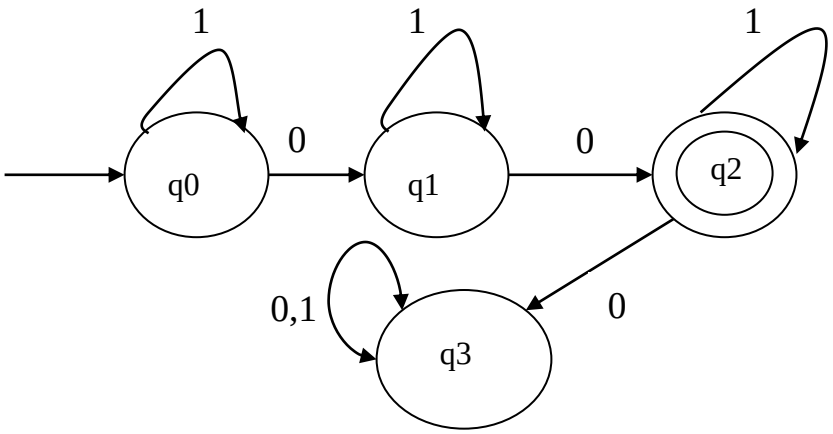
Note: DFA = Deterministic Finite Automata NFA = Nondeterministic Finite Automata
PDA = Push-Down Automata CFG=Context Free Grammar

Here are the pumping lemmas:

If A is a regular language, then there is a number p (the pumping length) where, if s is any string in A of length at least p, then s may be divided into 3 pieces, $s = xyz$, satisfying the following conditions:

1. For each $i \geq 0$, $xy^iz \in$ A,
2. $|y| > 0$, and
3. $|xy| \leq p$

1. Let M be the Deterministic Finite Automata (DFA) shown below



a. Provide a <u>formal</u> description of M below (6 points)

b. In plain English, describe the language described by this DFA. In order to get full credit, you need to provide reasonably succinct description that ignores irrelevant considerations. Fill in the blank below. (3 points)

The DFA shown above describes a language that _____.

c. Is the language described in part 1b a context-free language? Circle "Yes" or "No" and then briefly justify your answer. (3 points) Yes No.

2. Actual Factuals. Answer the following True/False and short answer questions.

   a. A DFA is equivalent in expressive power to an NFA. True False

   b. Deterministic and non-deterministic PDA's have equivalent expressive power. True False

   c. An NFA can recognize any language that a PDA can recognize True False

   d. You can build a PDA to recognize $\{0^{500}1^{40000} \cup 1^{1000}0^{200}\}$ True False

   e. The language 0^n0^n is regular True False

   f. You can convert an NFA with *n* states to a DFA with _____ states.

   g. The minimum pumping length of the string 0*1* is: _____

   h. The minimum pumping length of the string 00* is: _____

3. Assume an alphabet Σ that is {0, 1}

   a. Draw the *simplest* possible DFA (in terms of number of states and arcs) that describes the language of all strings that end in "00". (7 points)

   b. Draw the *simplest* possible NFA (in terms of number of states and arcs) that describes the language of all strings that end in "00". (7 points)

   c. Provide the regular expression that describes the language in part a. (3 points)

4. Draw the NFA that recognizes the language where w contains the substring 0101. Do this using 5 states and assuming a binary alphabet.

5. Your friend Brian is trying to prove that the language ww^{-R}, the language of palindromes, is not regular. For pumping length *p* he chooses the string $S = 01^p1^p0$, which is a palindrome. Can he use the pumping lemma for regular languages to prove that this language is not regular?

   Circle one: Yes No

   Now explain why below:

6. Prove or disprove the following statements. Assume that $\Sigma = \{0,1\}$

   a. 0^n1^{2n} is regular, $n \geq 1$

   . Circle one: Prove (statement true) Disprove (statement false)

   Prove or disprove below:

b. 0^n1^{2n} is context-free, $n \geq 1$

Circle one: Prove (statement true) Disprove (statement false)

Prove or disprove below:

7. Provide a Context Free grammar that generates the language 00*1*.

8. Provide a context free grammar that generates $L = \{a^nb^m: n \neq m\}$

9. Answer the following questions for the language of binary palindromes, L, where for any $w \in L$, $w=w^R$. (Note this allows for palindromes of any length, odd or even)

    a. Is L regular? No proof is required. Circle one: Yes No

    b. Is L context free? Circle one: Yes No

    c. Provide a proof for your answer in part b

    d. If L is context free, then provide an English language description of the PDA that recognizes L; if it is not context free, then explain why a PDA cannot be created to recognize L.