CISC 4090 Homework 1
(100 points total)

Note: according to our book on page 4, the set of natural numbers N does not include 0.

Question 1: (16 points) Examine the following "set-builder" descriptions of the following sets, and provide a list of set members. For full credit your members should show you capture the full diversity and range of the possible elements. Be especially careful for 1c.

Example "builder" description: $\{y \mid y=3x \text{ and } x \in \mathbb{N}\}$
Example answer: {3, 6, 9, 12, … }

a. $\{5m \mid m \in \mathbb{N} \text{ and } m>5\}$

b. $\{ \frac{x}{2} \mid x \in \mathbb{Z} \}$

c. $\{w \mid w$ is a string over the alphabet consisting of As and Bs, and w is a palindrome (reads the same forward and in reverse)$\}$

d. $\{(y, y\text{-}2) \mid y \in \mathbb{N}\}$

Question 2: (15 points) Provide a "set-builder" description (see Question 1) for each of the sets with elements listed below.

a. {10, 100, 1000, 10000, …}
b. {1, 4, 7, 10, 13, …}
c. {3, 4, 5, 6, 7, 8, …}
d. {1,2,3,4,5,6}
e. {}

Question 3: (20 points)
Let A={ab, aabb, aaabbb, aaaabbbb}, B={ab, abab, ababab}, and C={ab, aabb}

a. $C \subseteq A$ (circle one) True False
b. $B \subseteq A$ (circle one) True False
c. What is $B \cup C$?
d. What is $A \cap B$?
e. What is the power set of C?

Question 4: (10 points)

a. Given an arbitrary set A, with a total of |A| elements, how many elements are in the power set of A?

b. Given an arbitrary set A and B with |A| and |B| elements respectively, how many elements are in the "Cartesian product" of the two sets-- $A \times B$?

Question 5: (8 points) Let X be the set {2, 4, 6, 8, 10} and Y be the set {1, 2, 3, 4, 5}. The unary function $f: X \to Y$ and the binary function $g: X \times Y \to X$ are described in the following tables.

n	$f(n)$
2	3
4	3
6	5
8	5
10	1

g	1	2	3	4	5
2	4	4	4	4	4
4	6	6	6	10	10
6	10	8	6	4	2
8	2	2	6	6	6
10	4	8	10	8	4

a. What is the value of $f(8)$?
b. What are the range and domain of f?
c. What is the value of $g(8,4)$?
d. What is the value of $f(g(6,5))$?

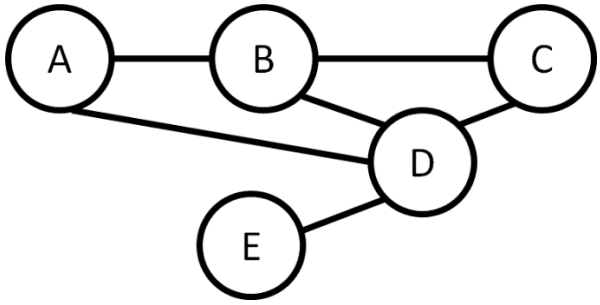
Question 6: (12 points) Consider the undirected graph $G=(V,E)$ where V, the set of nodes, is {1, 2, 3, 4} and E, the set of edges, is {{1,2}, {1,3}, {2,3}, {3,4}}.

a. Draw the graph G.

b. What are the degrees of each node?

c. Write a set of edges forming a path from node 3 to node 4 in the graph.

Question 7: (9 points) Write a formal description of the following graph.



Question 8: (10 points) Show that every graph with two or more nodes contains at least two nodes that have equal degrees.

Notes:

- We do not allow an edge from a node to itself.
- The graph does not have to be connected (some nodes may not have any edges)

Big Hint: Think of the pigeonhole principle, possibly taught in CISC 1100/1400. Feel free to look it up.