Daniel Leeds, R14, November 28, 2007
Final Exam: Mon. Dec 17, 8:30-11:30

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Various topics we've seen:
Recursion
Proofs: Lots of induction, proper proof style
Specifications
Continuations
Exceptions
Lazy programming
Modularity (structures and signatures)
Imperative programming
Concurrency
Type-checking
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From Fall 2006 Final:
2(i)
fun foldr f [ [] = z
| foldrfz(x::L)=f(x,foldrfzL)
fun ins ( $\mathrm{x},[\mathrm{l}$ ) $=[\mathrm{x}]$
ins $(x, y:: R)=$ if $x=y$ then $y:: R$ else $y:: i n s(x, R)$

We say L "has no repeats" if all its members are different.
Prove that, for all suitably typed lists $L$ and values $x$, if $L$ has no repeats then ins $(x, L)$ has no repeats. You can use the fact that the members of ins $(x, L)$ are $x$ and the members of L.

## 4

signature GRAPH $=$
sig
type "a graph
val build : ("a * "a) list -> "a graph
val roots : "a graph -> "a list
val delete : "a * "a graph -> "a graph
val isempty : "a graph -> bool
end;
complete:
structure Edges : GRAPH = struct
type "a graph = ("a * "a) list
fun build $\mathrm{L}=$
fun roots $\mathrm{L}=$

```
    fun delete =
    fun isempty L = null L
end;
```


## 6

datatype Token $=$ Left $\mid$ Right
E ::= <empty> | Left E1 Right E2
Write parse of type
parse : Token list -> (Token List -> bool) -> bool
such that
parse $\mathrm{L} k$ is true if there is a pair of lists L 1 and L 2 such that $\mathrm{L}=\mathrm{L} 1 @ \mathrm{~L} 2, \mathrm{~L} 1$ conforms to the grammar and $\mathrm{k}(\mathrm{L} 2)=$ true
parse $\mathrm{L} k$ returns false if there is no pair of lists $\mathrm{L} 1, \mathrm{~L} 2$ such that $\mathrm{L}=\mathrm{L} 1 @ \mathrm{~L} 2, \mathrm{~L} 1$ conforms to the grammar and $\mathrm{k}(\mathrm{L} 2)=$ true

Write balanced of type
balanced : Token list -> bool
Such that for all token lists L , balanced L returns true if L conforms to the grammar, returns false otherwise

## 9

Write simul of type
simul : 'a ref list * 'a list -> unit
such that for all $\mathrm{n}>=0$, all suitably typed refs $\mathrm{x}_{1}, \ldots, \mathrm{x}_{\mathrm{n}}$ and values $\mathrm{v}_{1}, \ldots, \mathrm{v}_{\mathrm{n}}$,
$\operatorname{simul}\left(\left[\mathrm{x}_{1}, \ldots, \mathrm{x}_{\mathrm{n}}\right],\left[\mathrm{v}_{1}, \ldots, \mathrm{v}_{\mathrm{n}}\right]\right)$ has the same effect as the sequence of assignments $\mathrm{x}_{1}:=\mathrm{v}_{1} ; \ldots$; $\mathrm{x}_{\mathrm{n}}:=\mathrm{v}_{\mathrm{n}}$. If two lists have unequal length, the function should raise the exception Unequal.

## 10

Write a recursive function
parfold : ('a * 'b -> 'b) -> 'b -> ('a chan * 'b chan) -> unit
such that, for suitably typed $\mathrm{F}, \mathrm{z}, \mathrm{a}$ and b , if channel a is supplied with the sequence $\mathrm{x}_{1}, \ldots, \mathrm{x}_{\mathrm{n}}, \ldots$ and b is a distinct channel, a thread executing parfold $\mathrm{Fz}(\mathrm{a}, \mathrm{b})$ will send z to be, receive $x_{1}$ from a, send $F\left(x_{1}, z\right)$ to $b$, receive $x_{2}$ from $a$, send $F\left(x_{2}, F\left(x_{1}, z\right)\right)$ to $b$, etc.. Do NOT use foldl. Do not store intermediate results.

