

Find the next term, find the recursive and closed forms for the sequences

3, 6, 9, 12, 15, __

0, 4, 8, 12, 16, __

5, 10, 20, 40, 80, 160

$$a_k = 2a_{k-1} \text{ for } k \geq 2$$

$$a_1 = 5$$

$$a_k = 5 \cdot 2^{k-1}$$

4, 9, 19, 39, 79, 159

$$a_k = 2a_{k-1} + 1 \text{ for } k \geq 2$$

$$a_1 = 4$$

$$a_k = 5 \cdot 2^{k-1} - 1$$

Recall the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, ...

With this in mind, find the next term and the recursive form for the following sequence

1, 1, 4, 10, 28, 76, 208, __

Express the following in Sigma notation

$$1+4+9+16=$$

$$3+3+3+3+3=$$

$$2+6+10+14+18+22=$$

$$10+15+20+25 = \sum_{i=2}^5 5i$$

Compute the value for the following sums

$$\sum_{j=2}^4 3j$$

$$\sum_{i=1}^5 (2i + 4)$$

$$\sum_{i=1}^5 2i + 4$$

$$\sum_{n=0}^3 n^3 = 0^3 + 1^3 + 2^3 + 3^3 = 0 + 1 + 8 + 27 = \mathbf{36}$$

Answer the following questions for each potential function:

Is this a function? If not, why not?

Is this an injection?

Is this a surjection?

Is this a bijection?

Is it invertible? If so, what is the inverse?

$$f: \{a,b,c,d\} \rightarrow \{q,r,s\}$$

X	a	b	c	D
f(x)	r	q	s	S

$$g: \{3,4,6,9\} \rightarrow \{1,3,5,7,9,12,15\}$$

X	3	4	6	9
g(x)	1	7	12	5

$$h: \mathbb{Z} \rightarrow \mathbb{Z} \quad h(y) = 5y - 2$$

Is function; Is injection; Is NOT surjection; Is NOT bijection; Is NOT invertible

$$n: \mathbb{Z} \rightarrow \mathbb{Z} \quad n(x) = \frac{x}{3}$$

$$m: \mathbb{R} \rightarrow \mathbb{R} \quad \text{where } |m(t)| = t$$

Is NOT a function, because multiple valid output values for positive input values, and no valid output values for negative input values.

Is NOT injection; Is NOT surjection; Is NOT bijection; Is NOT invertible.

Find the inverse for the following functions:

$$q: \mathbb{R} \rightarrow \mathbb{R} \quad q(t) = t - 5$$

$$q^{-1}: \mathbb{R} \rightarrow \mathbb{R} \quad q^{-1}(r) = r + 5$$

$$b: \mathbb{R} \rightarrow \mathbb{R} \quad b(x) = 4x + 3$$

$$c: \mathbb{Z} \rightarrow \mathbb{Z} \quad c(x) = x + 3$$

Use the Pigeonhole Principle to answer the following:

You are given a 6-sided die. How many times do you have to roll the die before it is guaranteed that you have rolled at least one of the numbers on the die at least two times?

In a crowded elevator at Fordham, can you be guaranteed that at least two people in the elevator were born in the same state of the United States (there are 50 states in the US, we assume everyone in the elevator was born in the US). Why or why not?

Compute the result of the function composition if possible, otherwise write "ill-defined."

$$f: \mathbb{N} \rightarrow \mathbb{N}, \quad g: \mathbb{Z} \rightarrow \mathbb{R}$$

$$f(x)=3x \quad g(x)=x^2$$

$$(f \circ g)(3) \quad \text{technically ill-defined, co-domain of } g(\mathbb{R}) \text{ is not subset of domain of } f(\mathbb{N})$$

I would give partial credit for $(f \circ g)(3) = f(g(3)) = f(3^2) = f(9) = 3 \times 9 = 27$

$$(g \circ f)(3)$$

$$(g \circ g)(2)$$

$$n: \mathbb{Z} \rightarrow \mathbb{N} \quad m: \mathbb{N} \rightarrow \mathbb{Q}$$

$$n(x) = |x+5| \quad m = \frac{x}{3}$$

$$(n \circ m)(x)$$

$$(n \circ n)(12)$$

$$\begin{aligned} n(n(12)) &= n(|12+5|) = n(17) \\ &= |17+5| = 22 \end{aligned}$$

$$(m \circ n)(y)$$

$$(m \circ m)(y)$$

$m \circ m$ ill-defined because co-domain of $m(\mathbb{Q})$ is not subset of domain of $m(\mathbb{N})$