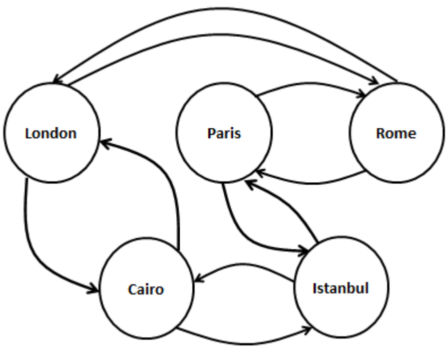
Example questions

For each of the three relations defined below:
- Draw a graph (circles and arrows) corresponding to the relation
- Say whether the relation is:
    + reflexive, irreflexive, neither
    + symmetric, anti-symmetric, neither
    + transitive, not-transitive

Relation 1, r_1, on the set of cities {London, Paris, Rome, Cairo, Istanbul}
r_1 = {(London, Rome), (London, Cairo), (Paris, Rome), (Paris, Istanbul), (Rome, London), (Rome, Paris), (Cairo, London), (Cairo, Istanbul), (Istanbul, Paris), (Istanbul, Cairo)}



irreflexive
symmetric
not transitive

Relation 2, r_2, on the set of people {Amy, Darren, Joanna, Larry, Veronica}
r_2 = {(Amy, Amy), (Amy, Joanna), (Amy, Veronica), (Darren, Darren), (Joanna, Joanna), (Joanna, Veronica), (Veronica, Veronica) }

Relation 3, r_3, on the set of numbers {1, 2, 3, 4, 5}
r_3 = {(1, 1), (1, 3), (1, 4), (2, 2), (3, 1), (3, 3), (4, 1), (4, 4), (5, 5)}

Write out the set of ordered pairs in the following relations on the integers $\mathbb{Z}$:
(x,y) is in the relation is and only if $x < 2y$

(x,y) is in the relation is and only if $3x-y=2$

(x,y) is in the relation is and only if $xy=3$ *(x times y equals 3)*

(x,y) is in the relation is and only if $4x=y$
{…, (-2,-8), (-1,-4), (0,0), (1,4), (2,8), …}

Consider the following relations on the set of all people and say whether the resulting relations are: reflexive, irreflexive, or neither; symmetric, anti-symmetric, or neither; transitive or not

Was born in the same city as

reflexive, symmetric, transitive

Is older than

Owns the same book as

Is the sibling of *("sibling" in this definition shares one or more parents)*

Answer the following questions for each potential function:

Is this a function? If not, why not?

Is this an injection?

Is this a surjection?

Is this a bijection?

Is it invertible? If so, what is the inverse?

f: {a,b,c,d} →{q,r,s}

x	a	b	c	d
f(x)	r	q	s	s

Is function; Is NOT injection; Is surjection; Is NOT bijection; Is NOT invertible

g: {3,4,6,9} → {1,3,5,7,9,12,15}

x	3	4	6	9
g(x)	1	7	12	5

h: $\mathbb{Z} \to \mathbb{Z}$ h(y) = 5y – 2

n: $\mathbb{Z} \to \mathbb{Z}$ n(x) = $\frac{x}{3}$

m: $\mathbb{R} \to \mathbb{R}$ where |m(t)| = t

Find the inverse for the following functions:

q: $\mathbb{R} \to \mathbb{R}$ q(t) = t – 5

b: $\mathbb{R} \to \mathbb{R}$ b(x) = 4x + 3

c: $\mathbb{Z} \to \mathbb{Z}$ c(x) = x + 3

c^{-1}: $\mathbb{Z} \to \mathbb{Z}$ c^{-1}(y) = y - 3

Use the Pigeonhole Principle to answer the following:

You are given a 6-sided die. How many times do you have to roll the die before it is guaranteed that you have rolled at least one of the numbers on the die at least two times?

In a crowded elevator at Fordham, can you be guaranteed that at least two people in the elevator were born in the same state of the United States (there are 50 states in the US, we assume everyone in the elevator was born in the US). Why or why not?

In a standard elevator there will not be room for more than 20 people. You cannot be guaranteed that at least two people in the elevator were born in the same state, because the number of people in the elevator ($|E| \leq 20$) will be less than the number of states ($|S|=50$). $|E|$ must be greater than $|S|$ to guarantee to people in E will be assigned to the same state S.

Compute the result of the function composition if possible, otherwise write "ill-defined."

$f: \mathbb{N} \rightarrow \mathbb{N}$, $g: \mathbb{Z} \rightarrow \mathbb{R}$

$f(x)=3x$ $g(x)=x^2$

$(f \circ g)(3)$

$(g \circ f)(3)$ $g(f(3)) = g(3x3) = g(9) = 9^2 = 81$

$(g \circ g)(2)$

$n: \mathbb{Z} \rightarrow \mathbb{N}$ $m: \mathbb{N} \rightarrow \mathbb{Q}$

$n(x)=|x+5|$ $m=\frac{x}{3}$

$(n \circ m)(x)$ $(m \circ n)(y) = m(n(y)) = m(|y+5|) = \frac{|y+5|}{3}$ $m \circ n: \mathbb{Z} \rightarrow \mathbb{Q}$

$(n \circ n)(12)$ $(m \circ m)(y)$