

Example questions

In how many ways can 1 gold, 1 silver, and 1 bronze medal be awarded to 10 contestants?

How many ways are there to select 3 class representatives from a class of 25 students?

$$C(25,3)=\frac{25 \times 24 \times 23}{3!}=\frac{25 \times 24 \times 23}{6}=25 \times 4 \times 23=230 \quad \mathbf{230}$$

You can borrow up to 3 DVDs from your friends, who owns 30 DVDs.

- How many ways are there to select 1 DVD?
- How many ways are there to select 2 DVDs?
- How many ways are there to select 3 DVDs?
- How many ways are there to select 3 or fewer DVDs?

There are 8 songs you can play at your party. How many ways are there to select a play list containing 5 of these songs (where the order matters)?

You are taking a multiple choice test where each question has 5 possible answers to choose from --- A, B, C, D, or E --- and there are 4 questions in total.

- How many possible ways are there to select your answers?
 $5 \times 5 \times 5 \times 5 = \mathbf{625}$
- How many ways are there to select answers if no letter can be used for more than one question?
- How many ways are there to select answers if the fourth answer must be the same as the first?
- How many ways are there to select answers if one letter is used for two of the answers and the remaining answers use distinct letters?

You order a computer science book as a gift for your best friend (computer science books make the best gifts). It is available

- in hardcover and in paperback
- new and used
- in first, second, and third edition
- for overnight, 2-day, and 5-day delivery

How many ways can you order the computer science book?

I am forming a Fordham Jeopardy team and need to select 6 members. I have 7 candidates from the Computer Science department, 9 from the Economics department, 10 from the English department, and 12 from the Theater department.

(Give the expression to find the answers, actual number is welcome but not required)

- How many ways can I choose from all the candidates?
- How many ways can I choose a team where all members are from the same department?

$$C(7,6)+C(9,6)+C(10,6)+C(12,6)=7 + \frac{9!}{6!3!} + \frac{10!}{6!4!} + \frac{12!}{6!6!} = 7 + \frac{9 \times 8 \times 7}{3 \times 2 \times 1} + \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} + \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = 7 + (3 \times 4 \times 7) + (10 \times 3 \times 7) + (11 \times 2 \times 3 \times 2 \times 7) = 7 + 84 +$$

$$210 + 616 = 917 + 924 = \mathbf{1149}$$

- How many ways can I choose a team with 2 computer science students, 2 economics students, and 2 theatre students?
- How many ways can I choose a team with no computer science majors?

$$C(38,6) = \frac{38!}{32!6!} = \frac{38 \times 37 \times 36 \times 35 \times 34 \times 33}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{38 \times 37 \times 1 \times 35 \times 34 \times 33}{5 \times 4} = \frac{38 \times 37 \times 7 \times 34 \times 33}{4} = 19 \times 37 \times 7 \times 17 \times 33 = \mathbf{2,760,681}$$

$$C(31,6) = \frac{31!}{25!6!} = \frac{31 \times 30 \times 29 \times 28 \times 27 \times 26}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = \mathbf{736,281}$$

Solve for:

$$P(3,1)$$

$$P(4,2)$$

$$P(5,2) = 5 \times 4 = \mathbf{20}$$

$$P(m,0)$$

$$C(5,3)$$

$$C(4,1)$$

$$C(4,4) = 4 \times 3 \times 2 \times 1 = \mathbf{24 = 1}$$

You are given an exam with every question having either the answer “true” or “false.” There are 10 questions on the exam and you guess randomly for each question. Assuming each answer is equally likely to be true or false:

- What is the probability you get exactly 5 questions correct?
- What is the probability you get fewer than 8 questions correct?

You have a jar of jelly beans in front of you containing 20 cherry, 10 orange, 15 lemon, 15 watermelon, and 10 grape flavor. You take 5 beans without looking.

(Give the expression to find the answers, actual number/fraction is welcome but not required):

- What is the probability you pick one of each flavor?
- What is the probability exactly 2 beans you picked are grape?
- What is the probability none of the picked beans are cherry?

$$\frac{P(60, 5)}{P(70, 5)} = \frac{60 \times 59 \times 58 \times 57 \times 56}{70 \times 69 \times 68 \times 67 \times 66}$$

$$\frac{P(50, 5)}{P(70, 5)} = \frac{50 \times 49 \times 48 \times 47 \times 46}{70 \times 69 \times 68 \times 67 \times 66}$$

We go to the casino and play a dice-rolling game. Each die is 6 sided and 3 die are rolled.

- What is the probability all 3 die roll higher than 4?

$$\text{Prob}(\text{first roll} > 4) \times \text{Prob}(\text{second roll} > 4) \times \text{Prob}(\text{third roll} > 4) = \frac{2}{6} \times \frac{2}{6} \times \frac{2}{6} = \frac{8}{216}$$

- What is the probability at least one 3 is rolled?
- What is the probability all 3 rolls are different?
- What is the probability the first 2 dice rolled add up to 2 or 5?
- What is the probability the first roll is even or greater than 4?
- What is the probability the second roll is a multiple of 2 or of 3?

$$\text{Prob}(\text{multiple of 2}) + \text{Prob}(\text{multiple of 3}) - \text{Prob}(\text{multiple of 2 and 3}) = \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

I flip a biased coin where it will land on heads with 2/3 probability.

- What is the probability of getting 2 tails in 2 flips?
- What is the probability of getting all heads in 4 flips?
- What is the probability of getting fewer than 2 tails in 4 flips?

$$\text{Prob}(\text{exactly 1 tail}) + \text{Prob}(\text{no tails}) =$$

$$\text{Prob}(T\text{HHH}) + \text{Prob}(HT\text{HH}) + \text{Prob}(H\text{HTH}) + \text{Prob}(HH\text{HT}) + \text{Prob}(HH\text{HH}) = 4 \left(\frac{2}{3} \times \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} \right) +$$

$$\left(\frac{2}{3} \right)^4 = 4 \times \frac{8}{81} + \frac{16}{81} = \frac{32+16}{81} = \frac{48}{81}$$