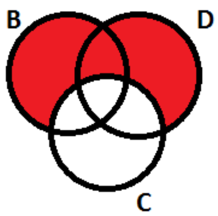
Example questions:

First, let us group Alice characters and objects into sets:

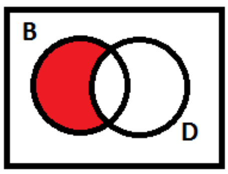
U = universal set of all Alice characters and objects
A={astronaut, T-Rex, AliceLiddell, hare, wizard, skeleton}
B={gazebo, cabin, sailboat, volcano}
C={AliceLiddell, hare, cat}
D={wizard, fairy, dragon}

What are the contents for each new set calculated below? When there is a *(V)* at the front of the question, draw a Venn diagram as well.
*(V)* B-A
P(D) {{},{wizard},{fairy},{dragon}, {wizard,fairy}, {wizard,dragon}, {fairy,dragon}, {wizard,fairy,dragon}}
CxB
*(V)* C∩A
*(V)* (B∪D)-C
{gazebo,cabin,sailboat,volcano, wizard, fairy, dragon}




P(A∩B)

Draw Venn diagrams for the following. You do not need to list the contents of the resulting sets:
(B∪C)∩A
C'
B∩D'



Now, let us group html tags into sets. *(You do not have to remember what these tags do to answer the questions; if you want, you can pretend they are just random words.)*

E={html, body, head}
F={{table, tr, td}, aHref, font, h1}
G={{head, title, meta}, {form, textarea, {input, radio}}}
H={h1, h2, h3, center, u}

Answer true or false. If false, explain why – for example, how you can change the equation to make it true.
|H|=5 **true**
|F|=6
{body, head} ⊆ E
meta ∈ G **false**, {head, title, meta} is an element of G
td ∈ F **false**, {table, tr, td} is an element of F
|G|=6
{html} ∈ E
|P(E)|=8

Give the elements based on the following set notation:
$J=\{y^2 : y\in \mathbb{N}\}$
$K=\{x : 3x\in \mathbb{Z}$ and $x\geq-2\}$
$L=\{y : |y|<4$ and $y\in \mathbb{Z} \}$
$M=\{z \mid z\in \mathbb{N}$ and $z^2=4\}$ **M={2}**

A class of 17 students takes a trip to Microsoft headquarters in Redmond, Washington. 10 students want to see the Xbox lab. 9 students want to see the secret teleportation lab. (Each student wants to see at least one of these two labs.) How many students want to go to both labs?
|All students|=|Xbox students|+|teleport students|-|Xbox-and-teleport students|
17 = 10 + 9 - |Xbox-and-teleport students|
|Xbox-and-teleport students|=19-17 = **2**

Each student in a CISC 1100 class will receive one or two prizes for the web site they wrote. The two prizes available are a new motorcycle and an old goat. There are 15 students in the class. 7 receive a goat and a motorcycle. 3 receive just a goat. How many students total receive a motorcycle (with or without the goat)?

We are constructing a world in the Alice programming environment and make several statements about this world. (*You do not have to remember anything about Alice programming to answer this question, but hopefully it will be more fun to think about if you do remember.*)
a = Alice is near the barn.
b =The barn door is closed.
c = A cow is coming out of the barn.
d = A penguin dances in front of Alice.

Write each of the following using the propositional variables a, b, c, and d.
Alice is near the barn and a penguin in not dancing in front of Alice.

If a cow is coming out of the barn and Alice is not near the barn, the barn door is not closed.

A cow is coming out of the barn if and only if a penguin dances in front of Alice. Furthermore, Alice is not near the barn and the penguin is not dancing in front of Alice.
$(c\leftrightarrow d) \wedge (a'\wedge d')$

Give the truth table of:
$a\wedge b'\rightarrow b$

| a | B | b' | a∧b' | a∧b'→b |
|---|---|---|---|---|
| T | T | F | F | **T** |
| T | F | T | T | **F** |
| F | T | F | F | **T** |
| F | F | T | F | **T** |

$(b \vee a)'$
$a \vee a'$
$b \oplus c$

Use truth tables to determine whether the following expression pairs are equivalent

$(p \wedge q)' \vee r \quad \equiv? \quad (q \rightarrow r) \vee p'$

$r \wedge p \oplus q \quad \equiv? \quad r \leftrightarrow q \vee p$

| p | Q | r | r∧p | r∧p⊕q | q∨p | r↔q∨p |
|---|---|---|---|---|---|---|
| T | T | T | T | **F** | T | **T** |
| T | T | F | F | **T** | T | **F** |
| T | F | T | T | **T** | T | **T** |
| T | F | F | F | **F** | T | **F** |
| F | T | T | F | **T** | T | **T** |
| F | T | F | F | **T** | T | **F** |
| F | F | T | F | **F** | F | **F** |
| F | F | F | F | **F** | F | **T** |

**not equivalent**

$r \wedge q \wedge (p \vee r') \quad \equiv? \quad r \wedge (p' \vee q')'$

Apply propositional laws to find equivalent expression:
*For example a∨a≡a using idempotent law (you don't have to name the law you are using)*
$(c \vee d)' \equiv c' \wedge d'$ using DeMorgan law
$[(b \rightarrow a)']'$
$(a \wedge b) \wedge c$

I have written my own version of the Alice programming environment called the "Daniel programming environment." There are five characters/objects in the Daniel world: a professor, a student, a robot, a rabbit, and a duck. I define several "functions". For each, answer:

- Is it a valid function?
- Is it an injection?
- Is it a surjection?
- Is it a bijection?
- Is it invertible?

$height_1$: {professor, student, robot, rabbit, duck} -> {1, 2, 3, 4, 5}

| Object | professor | student | robot | rabbit | Duck |
|---|---|---|---|---|---|
| $height_1$(object) | 5 | 4 | 1 | 3 | 2 |

Is a valid function, is injection, is surjection, is bijection, is invertible.

$height_2$: {professor, student, robot, rabbit, duck} -> $\mathbb{N}$

| object | professor | student | robot | rabbit | duck |
|---|---|---|---|---|---|
| $height_2$(object) | 5 | 4 | 1 | 3 | 2 |

$height_3$: {professor, student, robot, rabbit, duck} -> $\mathbb{N}$

| object | professor | student | robot | rabbit | duck |
|---|---|---|---|---|---|
| $height_3$(object) | 6.4 | 4.23 | .4 | 2.5 | 2 |

$height_4$: {professor, student, robot, rabbit, duck} -> {1, 4, 5}

| object | professor | student | robot | rabbit | duck |
|---|---|---|---|---|---|
| $height_4$(object) | 5 | 4 | 1 | 1 | 4 |

The duck in my Daniel world is always happier than the rabbit. I provide three happiness functions below. Find the inverse for each function. *(If the happiness example does not make sense to you, forget about the animals and just find the inverse.)*

$happy_1: \mathbb{R} \rightarrow \mathbb{R}$ $happy_1(x)=3x-2$

$happy_2: \mathbb{Z} \rightarrow \mathbb{Z}$ $happy_2(y)=y+4$
$happy_2^{-1}: \mathbb{Z} \rightarrow \mathbb{Z}$ $happy_2^{-1}(x)=x-4$

$happy_3: \mathbb{R}^{\geq 0} \rightarrow \mathbb{R}^{\geq 4}$ $happy_3(y)=(y+2)^2$

If we change the domain and co-domain for $happy_3$ so we now have:
$happy_4: \mathbb{Z} \rightarrow \mathbb{Z}$ $happy_4(y)=(y+2)^2$
the function is no longer invertible. Why is it no longer invertible? Specifically:

- Is it still an injection?
- Is it still a surjection?
- Is it still a bijection?

$happy_4$ is not an injection, $happy_4$ is not a surjection, $happy_4$ is not a bijection

I have a new student and I have taught her 5 html tags. I give her a lab assignment in which she must use at least 20 html tags. Presuming she only uses the tags I taught her, can I be guaranteed she will use at least one tag more than once? Explain why or why not using the Pigeonhole principle.

I have gotten a big new office with 12 windows out onto campus. 5 pigeons fly into my office (presumably to ask me about functions). Can I be guaranteed that at least two pigeons fly through the same window to get into my office? Explain why or why not using the Pigeonhole principle.

Compute the following function compositions. You may assume all compositions here are valid. (On the actual exam I may ask you whether a few examples are valid or ill-posed.)

$f: \mathbb{Z} \rightarrow \mathbb{N}$ $\quad f(x)=\begin{cases} x \text{ } if \text{ } x \geq 0 \\ 0 \text{ } if \text{ } x < 0 \end{cases}$

$g: \mathbb{R} \rightarrow \mathbb{R}$ $\quad g(y)=2y^2+2$

$g \circ f(-4)=g(f(-4))=g(0)=2\times 0^2+2=\mathbf{2}$
$g \circ f(8)$
$f \circ f(12)$
$g \circ g(1)$

$h: \mathbb{R} \rightarrow \mathbb{R}$ $\quad h(y)=y-5$
$k: \mathbb{R} \rightarrow \mathbb{R}$ $\quad k(x)=4x$
$h \circ k(y)=h(k(y))=h(4y)=\mathbf{4y-5}$
$k \circ k(y)$
$h \circ k(-3)$