

Example questions

For each of the three relations defined below:

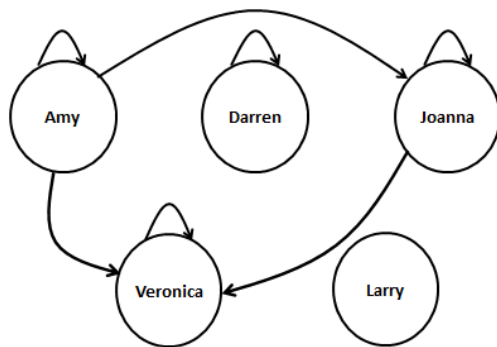
- Draw a graph (circles and arrows) corresponding to the relation
- Say whether the relation is:
 - + reflexive, irreflexive, neither
 - + symmetric, anti-symmetric, neither
 - + transitive, not-transitive

Relation 1, r_1 , on the set of cities {London, Paris, Rome, Cairo, Istanbul}

$$r_1 = \{(London, Rome), (London, Cairo), (Paris, Rome), (Paris, Istanbul), (Rome, London), (Rome, Paris), (Cairo, London), (Cairo, Istanbul), (Istanbul, Paris), (Istanbul, Cairo)\}$$

Relation 2, r_2 , on the set of people {Amy, Darren, Joanna, Larry, Veronica}

$$r_2 = \{(Amy, Amy), (Amy, Joanna), (Amy, Veronica), (Darren, Darren), (Joanna, Joanna), (Joanna, Veronica), (Veronica, Veronica)\}$$



not reflexive
anti-symmetric
transitive

Relation 3, r_3 , on the set of numbers {1, 2, 3, 4, 5}

$$r_3 = \{(1, 1), (1, 3), (1, 4), (2, 2), (3, 1), (3, 3), (4, 1), (4, 4), (5, 5)\}$$

Write out the set of ordered pairs in the following relations on the integers \mathbb{Z} :

(x, y) is in the relation and only if $x < 2y$

{...

..., (-5, -1), (-4, -1), (-3, -1),

..., (-3, 0), (-2, 0), (-1, 0),

..., (-1, 1), (0, 1), (1, 1),

... }

(x, y) is in the relation and only if $3x - y = 2$

(x,y) is in the relation is and only if $xy=3$ (x times y equals 3)

$\{(-3,-1), (-1,-3), (1,3), (3,1)\}$

(x,y) is in the relation is and only if $4x=y$

Consider the following relations on the set of all people and say whether the resulting relations are: reflexive, irreflexive, or neither; symmetric, anti-symmetric, or neither; transitive or not

Was born in the same city as

Is older than

Owens the same book as

Is the sibling of (*"sibling" in this definition shares one or more parents*)

Irreflexive, symmetric, not transitive

Answer the following questions for each potential function:

Is this a function? If not, why not?

Is this an injection?

Is this a surjection?

Is this a bijection?

Is it invertible? If so, what is the inverse?

$f: \{a,b,c,d\} \rightarrow \{q,r,s\}$

X	a	b	c	D
f(x)	r	q	s	S

$g: \{3,4,6,9\} \rightarrow \{1,3,5,7,9,12,15\}$

X	3	4	6	9
g(x)	1	7	12	5

$h: \mathbb{Z} \rightarrow \mathbb{Z} \quad h(y) = 5y - 2$

Is function; Is injection; Is NOT surjection; Is NOT bijection; Is NOT invertible

$n: \mathbb{Z} \rightarrow \mathbb{Z} \quad n(x) = \frac{x}{3}$

$m: \mathbb{R} \rightarrow \mathbb{R} \quad \text{where } |m(t)| = t$

Is NOT a function, because multiple valid output values for positive input values, and no valid output values for negative input values.

Is NOT injection; Is NOT surjection; Is NOT bijection; Is NOT invertible.

Find the inverse for the following functions:

$$q: \mathbb{R} \rightarrow \mathbb{R} \quad q(t) = t - 5$$

$$q^{-1}: \mathbb{R} \rightarrow \mathbb{R} \quad q^{-1}(r) = r + 5$$

$$b: \mathbb{R} \rightarrow \mathbb{R} \quad b(x) = 4x + 3$$

$$c: \mathbb{Z} \rightarrow \mathbb{Z} \quad c(x) = x + 3$$

Use the Pigeonhole Principle to answer the following:

You are given a 6-sided die. How many times do you have to roll the die before it is guaranteed that you have rolled at least one of the numbers on the die at least two times?

In a crowded elevator at Fordham, can you be guaranteed that at least two people in the elevator were born in the same state of the United States (there are 50 states in the US, we assume everyone in the elevator was born in the US). Why or why not?

Compute the result of the function composition if possible, otherwise write "ill-defined."

$$f: \mathbb{N} \rightarrow \mathbb{N}, \quad g: \mathbb{Z} \rightarrow \mathbb{R}$$

$$f(x) = 3x \quad g(x) = x^2$$

$(f \circ g)(3)$ technically ill-defined, co-domain of g (\mathbb{R}) is not subset of domain of f (\mathbb{N})

I would give partial credit for $(f \circ g)(3) = f(g(3)) = f(3^2) = f(9) = 3 \times 9 = 27$

$$(g \circ f)(3)$$

$$(g \circ g)(2)$$

$$n: \mathbb{Z} \rightarrow \mathbb{N} \quad m: \mathbb{N} \rightarrow \mathbb{Q}$$

$$n(x) = |x+5| \quad m = \frac{x}{3}$$

$$(n \circ m)(x)$$

$$(n \circ n)(12)$$

$$n(n(12)) = n(|12+5|) = n(17) \\ = |17+5| = 22$$

$$(m \circ n)(y)$$

$$(m \circ m)(y)$$

$m \circ m$ ill-defined because co-domain of m (\mathbb{Q}) is not subset of domain of m (\mathbb{N})