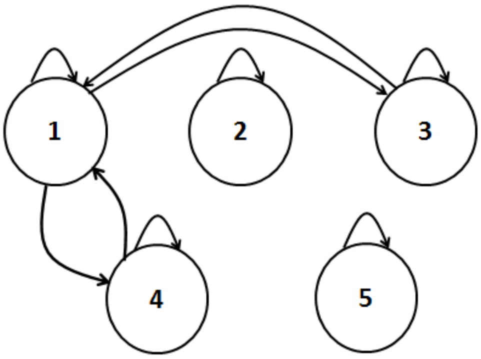
Example questions

For each of the three relations defined below:
- Draw a graph (circles and arrows) corresponding to the relation
- Say whether the relation is:
    + reflexive, irreflexive, neither
    + symmetric, anti-symmetric, neither
    + transitive, not-transitive

Relation 1, r_1, on the set of cities {London, Paris, Rome, Cairo, Istanbul}
   r_1 = {(London, Rome), (London, Cairo), (Paris, Rome), (Paris, Istanbul), (Rome, London), (Rome, Paris), (Cairo, London), (Cairo, Istanbul), (Istanbul, Paris), (Istanbul, Cairo)}

Relation 2, r_2, on the set of people {Amy, Darren, Joanna, Larry, Veronica}
   r_2 = {(Amy, Amy), (Amy, Joanna), (Amy, Veronica), (Darren, Darren), (Joanna, Joanna), (Joanna, Veronica), (Veronica, Veronica) }

Relation 3, r_3, on the set of numbers {1, 2, 3, 4, 5}
   r_3 = {(1, 1), (1, 3), (1, 4), (2, 2), (3, 1), (3, 3), (4, 1), (4, 4), (5, 5)}



reflexive
symmetric
not transitive

Write out the set of ordered pairs in the following relations on the integers $\mathbb{Z}$:
(x,y) is in the relation is and only if $x < 2y$

(x,y) is in the relation is and only if $3x-y=2$
{…, (-1, -5), (0,-2), (1,1), …}

(x,y) is in the relation is and only if $xy=3$ *(x times y equals 3)*

(x,y) is in the relation is and only if $4x=y$

Consider the following relations on the set of all people and say whether the resulting relations are: reflexive, irreflexive, or neither; symmetric, anti-symmetric, or neither; transitive or not

Was born in the same city as

Is older than

irreflexive, anti-symmetric, transitive

Owns the same book as

reflexive, symmetric, not-transitive

Is the sibling of *("sibling" in this definition shares one or more parents)*

Answer the following questions for each potential function:

Is this a function? If not, why not?

Is this an injection?

Is this a surjection?

Is this a bijection?

Is it invertible? If so, what is the inverse?

f: {a,b,c,d} →{q,r,s}

X	A	b	c	D
f(x)	R	q	s	S

g: {3,4,6,9} → {1,3,5,7,9,12,15}

X	3	4	6	9
g(x)	1	7	12	5

Is function; Is injection; Is NOT surjection; Is NOT bijection; Is NOT invertible

h: $\mathbb{Z} \to \mathbb{Z}$ h(y) = 5y – 2

n: $\mathbb{Z} \to \mathbb{Z}$ n(x) = $\frac{x}{3}$

Is NOT function, because inputs such as 2 or 7 are not mapped to integer outputs when divided by 3.

Is NOT injection; Is NOT surjection; Is NOT bijection; Is NOT invertible

m: $\mathbb{R} \to \mathbb{R}$ where |m(t)| = t

Find the inverse for the following functions:

q: $\mathbb{R} \to \mathbb{R}$ q(t) = t – 5

b: $\mathbb{R} \to \mathbb{R}$ b(x) = 4x + 3

$b^{-1}: \mathbb{R} \rightarrow \mathbb{R}$ $b^{-1}(y) = \frac{y-3}{4}$

c: $\mathbb{Z} \rightarrow \mathbb{Z}$ c(x) = x + 3

Use the Pigeonhole Principle to answer the following:
You are given a 6-sided die. How many times do you have to roll the die before it is guaranteed that you have rolled at least one of the numbers on the die at least two times?
You need to roll the die at least 7 times, so the number of rolls |R| is greater than the number of die-sides |S|=6.

In a crowded elevator at Fordham, can you be guaranteed that at least two people in the elevator were born in the same state of the United States (there are 50 states in the US, we assume everyone in the elevator was born in the US). Why or why not?

Compute the result of the function composition if possible, otherwise write "ill-defined."

f: $\mathbb{N} \rightarrow \mathbb{N}$, g: $\mathbb{Z} \rightarrow \mathbb{R}$
f(x)=3x g(x)=x^2
(f∘g)(3)
(g∘f)(3)
(g∘g)(2) Ill-defined because the co-domain of g ($\mathbb{R}$) is not a subset of the domain of g ($\mathbb{Z}$)
Partial credit for (g∘g)(2)=g(g(2))=g(2^2)=g(4)=4^2=16

n: $\mathbb{Z} \rightarrow \mathbb{N}$ m: $\mathbb{N} \rightarrow \mathbb{Q}$
n(x)=|x+5| m=$\frac{x}{3}$
(n∘m)(x) Ill-defined because the co-domain of m ($\mathbb{Q}$) is not a subset of the domain of n ($\mathbb{Z}$)
Partial credit for (n∘m)(x)=n(m(x))=n($\frac{x}{3}$)=|$\frac{x}{3}$+5|, but not actually valid
(m∘n)(y)
(n∘n)(12) (m∘m)(y)