CISC 3250  
Systems Neuroscience  
Representations in the brain  

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JMH 332

Computational representations describing a visual object

• A picture is worth a million pixels
  – Digital picture broken into a grid of boxes – pixels
  – Each pixel contains a color

• Translate from pixels to category label:
  floss  flour  flower  flume  flute  foam

Computer storage

Memory for data

• Information stored as billions of numbers (giga-bytes)
• Groups of numbers stored in sequence represent single concept
  – flower  1000 x 1000 x 3 matrix
• Each piece of information has location in memory
  – flower starts at address 100,000,5000
  – song1 starts at address 103,000,5000

Data in the brain

• Neural location related to information encoded

• Progression of encoding for increasingly complex concepts
Simple outline of vision pathway

1. Retina: pixel detectors
2. Primary visual cortex (V1): edge detectors
3. Second-cortical layer (V2?): edge combination detectors
   ... 
4. Higher-cortical layer: Full-object detectors

Interacting representations: feedforward network

- More-complex information/features computed from simpler information/features

Cortical organization and feature organization

- Nearby neurons respond to similar features
- Neuron can respond with intermediate rates to features deviating from maximum preference

Can be supported by local excitation

Lateral connections: surround suppression

Neuron can have suppressed response for features deviant from maximum preference

Typical V1 receptive field

Suppression/competition with interneurons

- In common cortical circuits, there are feedforward excitatory inputs and lateral inhibitory inputs
- Relative weighting achieves balance between activation and suppression

The pathway for smell processing

- Olfactory Epithelium (OE) – input
- Mitral – output
- PGe – lateral inhibitor

Simplified circuit
Competition on behavior level

Opposing interpretations of scene

Classes of representation

Local representation
- Neural level: “grandmother” cell
- “Region” level: face region, place region

<table>
<thead>
<tr>
<th>Parahippocampal place area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fusiform face area</td>
</tr>
<tr>
<td>Visual word form area</td>
</tr>
<tr>
<td>Lateral occipital cortex (shapes)</td>
</tr>
</tbody>
</table>

Fully distributed representation
- Every neuron/region plays a part

Sparsely-distributed representation
- Neural level: hyper-column for perceptual feature

- “Region” level: face network in medial temporal, lateral temporal, anterior parietal

Data in the brain

- Neural location related to information encoded
- Progression of encoding for increasingly complex concepts
Principles of information coding: binary

How many things can we represent with $n$ binary ($g_{\text{step}}$ activation function) neurons?

- Complete sparse coding: $n$ things
  - Firing: banana, apple, pear
  - Not firing: lime, orange, lemon, no fruit

- Complete distributed coding: $2^n$ things
  - Firing: banana, apple, pear
  - Not firing: blueberry, orange, apple, pear

Biology of sparse coding

- Preserving energy – higher spiking rate requires higher energy
- Representational fan-out
  - ~1 million neurons in retina -> ~140 million neurons in V1 (primary visual cortex)
  - ~50,000 neurons in cochlea -> 1.6 million neurons in A1 (primary auditory cortex)

Coding on a scale: sparsity

Typically we will say “sparsity” is using at most 10% of available neurons

Coding on a scale: distributed + overlapping

What does this encode?
Coding on a scale: distributed + overlapping
Responses for each property add together

<table>
<thead>
<tr>
<th>Mood (sad - happy)</th>
<th>Age (0 - 100)</th>
<th>Amount of hair (bald - long)</th>
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</thead>
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<td>1 - 1 Hz - sad</td>
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<td>1 - 1 Hz - bald</td>
</tr>
<tr>
<td>50 - 50 Hz - happy</td>
<td>50 - 50 Hz - old</td>
<td>50 - 50 Hz - full-hair</td>
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</tbody>
</table>

How do we encode: happy (100%), mid-age (50%), light hair (1%)?

\[ \sum_j \text{level}_j \text{pattern}_j \]

Responses for each property add together

Coding on a scale: distributed + overlapping
Responses for each property add together

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</table>

How do we encode: sad (5%), mid-age (50%), hairy (100%)?

\[ \sum_j \text{level}_j \text{pattern}_j \]

Responses for each property add together
Coding on a scale: distributed + overlapping

Responses for each property add together

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What does this encode? 0 20 40
What does this encode? 50 20 75

Decoding large neural codes

Information from neuron patterns
• Happy
• Old
• Hairy
• Loud

Overlay of multiple patterns and noise
• What property is this?

Decoding with tuning curves

Use spiking rates from multiple neurons to determine encoded feature
• 15 Hz firing rate for red neuron means sound 400 or 800 Hz (at 10 dB)
• 15 Hz for red and 6 Hz for blue requires sound 800 Hz (at 10 dB)

Actual decoding incorporates noise/natural variability in spiking
Population coding to find direction of motion

Non-normalized population coding

\[ s_{dir} = \sum_{i} r_i s_i^{pref} \]

<table>
<thead>
<tr>
<th>r</th>
<th>1</th>
<th>4</th>
<th>1</th>
<th>0</th>
</tr>
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<tbody>
<tr>
<td>s^{pref}</td>
<td>[0]</td>
<td>[1]</td>
<td>[0]</td>
<td>[-1]</td>
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\[ s_{dir} = \]

Population coding to find direction of motion

“Normalized” firing rate

\[ \hat{r}_i = \frac{r_i - r_i^{min}}{r_i^{max} - r_i^{min}} \]

If \( r_i^{min} = 1, r_i^{max} = 6 \) for

Then \( \hat{r}_i = \frac{4-1}{6-1} = \frac{3}{5} = 0.6 \)

Normalized \( \hat{r} \) will always be between 0 and 1

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Normalized firing rates

\( r_{min}=0 \) Hz, \( r_{max}=60 \) Hz

<table>
<thead>
<tr>
<th>r</th>
<th>30</th>
<th>30</th>
<th>10</th>
<th>0</th>
</tr>
</thead>
<tbody>
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Normalized firing rates
Another example
Assume for all neurons \( r_{\text{min}} = 10 \text{ Hz}, r_{\text{max}} = 100 \text{ Hz} \)

Population coding to find direction of motion

**“Normalized” pop’n coding**

\[
\hat{s}_{\text{pop}} = \frac{\sum_i \hat{r}_i s_{i}^{\text{pref}}}{\sum_j \hat{r}_j}
\]

\[
\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}
\]

\[
\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.5 \end{bmatrix} + 0.05 + 0.05 + 0.05 + 0.05 = \begin{bmatrix} 0.83 \end{bmatrix}
\]

Linear algebra

- **Left matrix:** data
  - Rows: different data points
  - Columns: different features
- **Right matrix:** column contains weights for weighted sum
Matrices and weighted sums

\[
\begin{bmatrix}
1 & 4 & 1 & 0 \\
0 & -1 & 1 & -1
\end{bmatrix}
\]

\[
1 \begin{bmatrix}
0 \\
-1
\end{bmatrix} + 4 \begin{bmatrix}
1 \\
0
\end{bmatrix} + 1 \begin{bmatrix}
0 \\
1
\end{bmatrix} + 0 \begin{bmatrix}
-1 \\
0
\end{bmatrix} = \begin{bmatrix}
4 \\
0
\end{bmatrix}
\]

Matrix multiplication:

Sum \{left \text{row} \times right \text{column}\}

\[
\begin{bmatrix}
a & b & c \\
d & e & f
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
a x + b y + c z \\
d x + e y + f z
\end{bmatrix}
\]