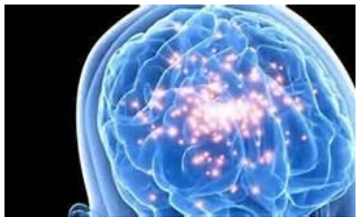
CISC 3250
Systems Neuroscience

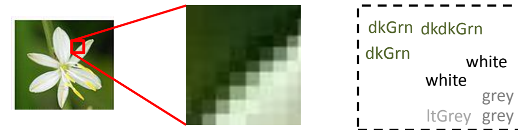
Representations in the brain



Professor Daniel Leeds
dleeds@fordham.edu
JMH 332

Computational representations describing a visual object

- A picture is worth a million pixels
  - Digital picture broken into a grid of boxes – pixels
  - Each pixel contains a color



- Translate from pixels to category label:

floss flour flower flume flute foam

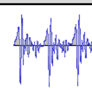
4

Computer storage

Memory for data
- Information stored as billions of numbers (giga-bytes)
- Groups of numbers stored in sequence represent single concept
  - flower 1000 x 1000 x 3 matrix
- Each piece of information has location in memory
  - flower starts at address 100,000,5000
  - song1 starts at address 103,000,5000

Memory

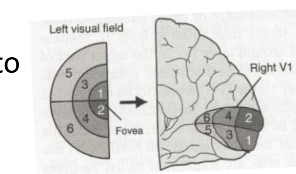
⋮
flower: 1,1,red
1,1,green
1,1,blue
1,2,red
1,2,green
1,2,blue
⋮
1000,1000,red
1000,1000,green
1000,1000,blue
song1: sound at 0ms
sound at 10ms
sound at 20ms



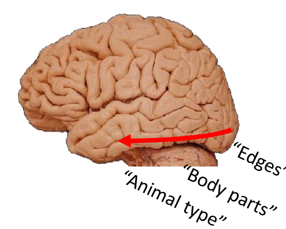
5

Data in the brain

- Neural location related to information encoded



- Progression of encoding for increasingly complex concepts



6

Simple outline of vision pathway

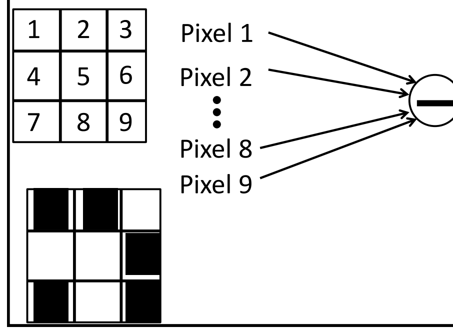
1. Retina: pixel detectors
2. Primary visual cortex (V1): edge detectors
3. Second-cortical layer (V2?): edge combination detectors

. . .

N. Higher-cortical layer: Full-object detectors

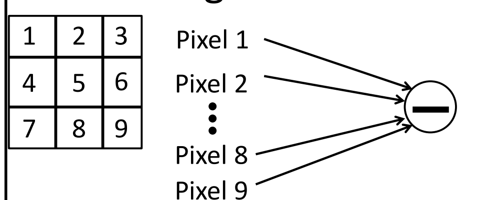
Interacting representations: feedforward network

- More-complex information/features computed from simpler information/features

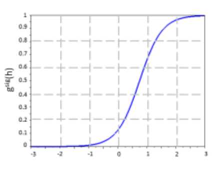


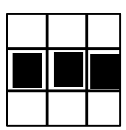
w_1=-.5
w_2=-1
w_3=-.5
w_4=.5
w_5=1
w_6=.5
w_7=-.5
w_8=-1
w_9=-.5

Edge detector in action



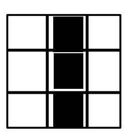
w_1=-.5 w_7=-.5
w_2=-1 w_8=-1
w_3=-.5 w_9=-.5
w_4=.5
w_5=1
w_6=.5





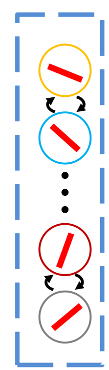
p_1=0, p_2=0, p_3=0,
p_4=1, p_5=1, p_6=1,
p_7=0, p_8=0, p_9=0

h=.5+1+.5=2
g(2)=0.95

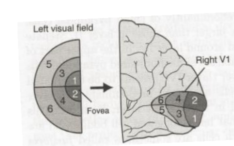


p_1=0, p_2=1, p_3=0,
p_4=0, p_5=1, p_6=0,
p_7=0, p_8=1, p_9=0

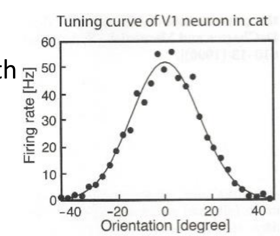
Cortical organization and feature organization



Nearby neurons respond to similar features



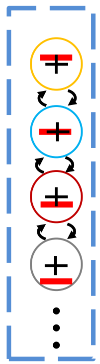
Neuron can respond with intermediate rates to features deviating from maximum preference



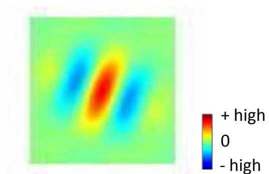
Henry et al., *J Neurophys* 1974.

Can be supported by local excitation

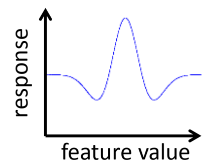
Lateral connections: surround suppresion



Neuron can have suppressed response for features deviant from maximum preference



Typical V1 receptive field



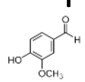
12

Suppression/competition with interneurons

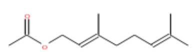
- In common cortical circuits, there are feedforward excitatory inputs and lateral inhibitory inputs
- Relative weighting achieves balance between activation and suppression

13

The pathway for smell processing



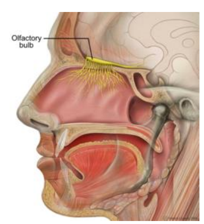
vanilla



rose

Nose/olfactory epithelium → Olfactory bulb (in cortex) → Orbitofrontal cortex, Amygdala, Hippocampus

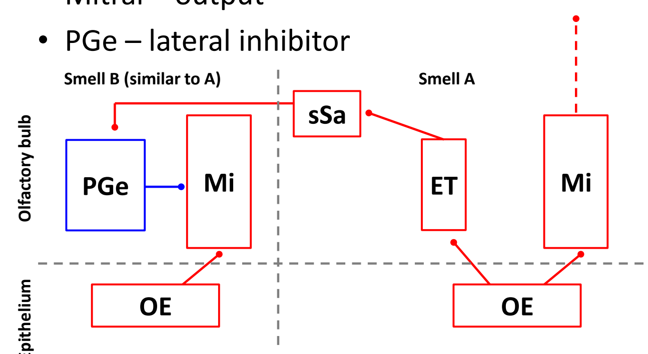
Chemical shape detection



14

Simplified circuit

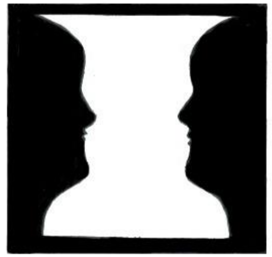
- Olfactory Epithelium (OE) – input
- Mitral – output
- PGe – lateral inhibitor



16

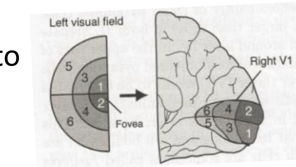
Competition on behavior level

Opposing interpretations of scene

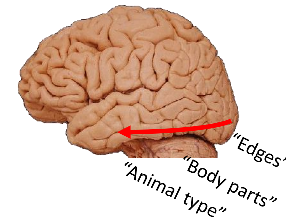


Data in the brain

- Neural location related to information encoded



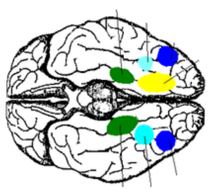
- Progression of encoding for increasingly complex concepts



Classes of representation

Local representation

- Neural level: "grandmother" cell
- "Region" level: face region, place region



Parahippocampal place area

Fusiform face area

Visual word form area

Lateral occipital cortex (shapes)

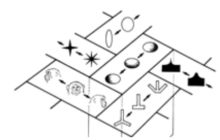
Classes of representation

Fully distributed representation

- Every neuron/region plays a part

Sparsely-distributed representation

- Neural level: hyper-column for perceptual feature



Tanaka 2003, columns of neurons for shape types in IT

- "Region" level: face network in medial temporal, lateral temporal, anterior parietal

Principles of information coding: binary

How many things can we represent with *n* binary (g^{step} activation function) neurons?

- Complete sparse coding: n things

firing

not firing

banana apple pear

- Complete distributed coding: 2^n things

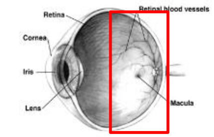
banana blueberry

orange apple pear

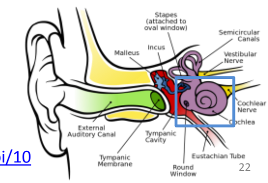
lime lemon No fruit

Biology of sparse coding

- Preserving energy – higher spiking rate requires higher energy
- Representational fan-out
  - ~1 million neurons in retina -> ~140 million neurons in V1 (primary visual cortex)



  - ~50,000 neurons in cochlea -> 1.6 million neurons in A1 (primary auditory cortex)



http://www.plosbiology.org/article/info:doi/10.1371/journal.pbio.0030137

Coding on a scale: sparsity

high firing

mid firing

not firing

sad ambi-valent happy

mood (sad – happy)

young mid-age old

age (0 – 100)

bald mid-hair hairy

amount hair (bald – long)

Typically we will say "sparsity" is using at most 10% of available neurons

Coding on a scale: distributed + overlapping

high firing mid firing not firing

sad ambi-valent happy

mood (sad – happy)

young mid-age old

age (0 – 100)

bald mid-hair hairy

amount hair (bald – long)

What does this encode?

Coding on a scale: distributed + overlapping

Responses for each property add together

1 - 1 Hz – sad	- 1 1 Hz – young	- - 1 Hz – bald
25 – 25 Hz – neutral	- 25 25 Hz – middle	- - 25 Hz – middle
50 – 50 Hz – happy	- 50 50 Hz – old	- - 50 Hz – full-hair
mood (sad – happy)	age (0 – 100)	amount hair (bald – long)

How do we encode: happy (100%), mid-age (50%), light hair (1%)?

$$\sum_j level_j\ pattern_j$$

Coding on a scale: distributed + overlapping

Responses for each property add together

1 – 1 Hz – sad	- 1 1 Hz – young	- - 1 Hz – bald
25 – 25 Hz – neutral	- 25 25 Hz – middle	- - 25 Hz – middle
50 – 50 Hz – happy	- 50 50 Hz – old	- - 50 Hz – full-hair
mood (sad – happy)	age (0 – 100)	amount hair (bald – long)

How do we encode: happy (100%), mid-age (50%), light hair (1%)?

$$\sum_j level_j\ pattern_j$$

n1	n2	n3	
50	0	50	happy
0	25	25	mid-age
<u>0</u>	<u>0</u>	<u>0.5</u>	light hair
50	**25**	**75.5**	

Coding on a scale: distributed + overlapping

Responses for each property add together

1 – 1 Hz – sad	- 1 1 Hz – young	- - 1 Hz – bald
25 – 25 Hz – neutral	- 25 25 Hz – middle	- - 25 Hz – middle
50 – 50 Hz – happy	- 50 50 Hz – old	- - 50 Hz – full-hair
mood (sad – happy)	age (0 – 100)	amount hair (bald – long)

How do we encode: sad (5%), mid-age (50%), hairy (100%)?

$$\sum_j level_j\ pattern_j$$

Coding on a scale: distributed + overlapping

Responses for each property add together

1 – 1 Hz – sad	- 1 1 Hz – young	- - 1 Hz – bald
25 – 25 Hz – neutral	- 25 25 Hz – middle	- - 25 Hz – middle
50 – 50 Hz – happy	- 50 50 Hz – old	- - 50 Hz – full-hair
mood (sad – happy)	age (0 – 100)	amount hair (bald – long)

How do we encode: sad (5%), mid-age (50%), hairy (100%)?

$$\sum_j level_j\ pattern_j$$

n1	n2	n3	
2.5	0	2.5	happy
0	25	25	mid-age
<u>0</u>	<u>0</u>	<u>50</u>	light hair
2.5	**25**	**77.5**	

Coding on a scale: distributed + overlapping

Responses for each property add together

mood (sad – happy)	age (0 – 100)	amount hair (bald – long)
1 – 1 Hz – sad	- 1 1 Hz – young	- - 1 Hz – bald
25 – 25 Hz – neutral	- 25 25 Hz – middle	- - 25 Hz – middle
50 – 50 Hz – happy	- 50 50 Hz – old	- - 50 Hz – full-hair

What does this encode? 0 20 40

What does this encode? 50 20 75

Coding on a scale: distributed + overlapping

Responses for each property add together

mood (sad – happy)	age (0 – 100)	amount hair (bald – long)
1 – 1 Hz – sad	- 1 1 Hz – young	- - 1 Hz – bald
25 – 25 Hz – neutral	- 25 25 Hz – middle	- - 25 Hz – middle
50 – 50 Hz – happy	- 50 50 Hz – old	- - 50 Hz – full-hair

What does this encode? 0 20 40 (Neuron 1, Neuron 2, Neuron 3)

Very sad: contributes: 0 x [50 0 50] = 0 0 0

Middle-age: contributes .4 x [0 50 50] = 0 20 20

Middle-hair: contributes .4 x [0 0 50] = 0 0 20

<u>Summing together:</u> 0 20 40

Coding on a scale: distributed + overlapping

Responses for each property add together

mood (sad – happy)	age (0 – 100)	amount hair (bald – long)
1 – 1 Hz – sad	- 1 1 Hz – young	- - 1 Hz – bald
25 – 25 Hz – neutral	- 25 25 Hz – middle	- - 25 Hz – middle
50 – 50 Hz – happy	- 50 50 Hz – old	- - 50 Hz – full-hair

What does this encode? 50 20 75 (Neuron 1, Neuron 2, Neuron 3)

Very happy: contributes 1 x [50 0 50] = 50 0 50

Middle-age: contributes .4 x [0 50 50] = 0 20 20

Bald: contributes .1 x [0 0 50] = 0 0 5

<u>Summing together:</u> 50 20 75

Decoding large neural codes

Information from neuron patterns

- Happy 
- Old 
- Hairy 
- Loud 

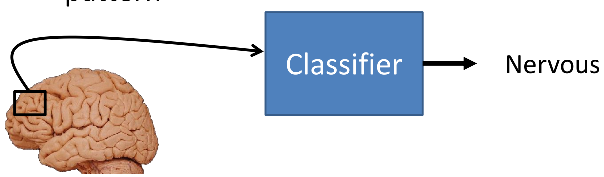
Overlay of multiple patterns and noise

- What property is this?

Decoding large neural codes

Classifier:

- If consistent response, can learn pattern
- If irrelevant response, cannot learn helpful pattern



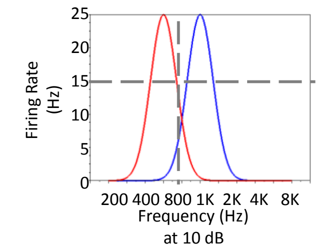
Method:

- 500 trials – measure mood, record brain responses
- Make classifier from neural patterns in trials 1-250
- Find accuracy to predict mood in trials 251-500

35

Decoding with tuning curves

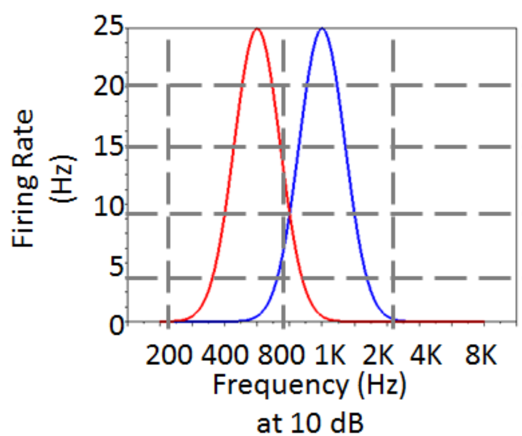
Use spiking rates from multiple neurons to determine encoded feature



- 15 Hz firing rate for red neuron means sound 400 or 800 Hz (at 10 dB)
- 15 Hz for red and 6 Hz for blue requires sound 800 Hz (at 10 dB)

Actual decoding incorporates noise/natural variability in spiking

36



37

Population coding to find direction of motion

Non-normalized population coding

- $s_{dir} = \sum_i r_i s_i^{pref}$

r	1	4	1	0
s^{pref} $\begin{bmatrix} x \\ y \end{bmatrix}$	$\begin{bmatrix} 0 \\ -1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$s_{dir} =$

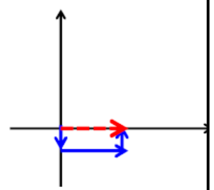
39

Population coding to find direction of motion

Non-normalized population coding

- $s_{dir} = \sum_i r_i s_i^{pref}$

r	1	4	1	0
	↓	→	↑	←
s^{pref} $\begin{bmatrix} x \\ y \end{bmatrix}$	$\begin{bmatrix} 0 \\ -1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \end{bmatrix}$



$$\begin{bmatrix} x \\ y \end{bmatrix} = 1\begin{bmatrix} 0 \\ -1 \end{bmatrix} + 4\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0\begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

41

Population coding to find direction of motion

"Normalized" firing rate

- $\hat{r}_i = \frac{r_i - r_i^{min}}{r_i^{max} - r_i^{min}}$

r	4
	→
s^{pref} $\begin{bmatrix} x \\ y \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

If $r^{min} = 1, r^{max} = 6$ for →

Then $\hat{r}_i = \frac{4-1}{6-1} = \frac{3}{5} = 0.6$

Normalized $\hat{r}$ will always be between 0 and 1

43

Normalized firing rates

r^{min}=0 Hz, r^{max}=60 Hz

r	30	30	10	0
	↓	→	↑	←
s^{pref} $\begin{bmatrix} x \\ y \end{bmatrix}$	$\begin{bmatrix} 0 \\ -1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

44

Normalized firing rates

r^{min}=0 Hz, r^{max}=60 Hz

	0.5	0.5	0.16	0
r	30	30	10	0
	↓	→	↑	←
s^{pref} $\begin{bmatrix} x \\ y \end{bmatrix}$	$\begin{bmatrix} 0 \\ -1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} \mathbf{0.5} \\ \mathbf{0} \end{bmatrix}$$

45

Population coding to find direction of motion

"Normalized" pop'n coding

For $\hat{s}_{pop}$, divide normalized rate by sum of all rates in neural population: $\sum_j \hat{r}_j$

- $\hat{s}_{pop} = \sum_i \frac{\hat{r}_i}{\sum_j \hat{r}_j} s_i^{pref}$

	$\hat{r}$	0.05	0.5	0.05	0
		↓	→	↑	←
s^{pref}	$\begin{bmatrix} x \\ y \end{bmatrix}$	$\begin{bmatrix} 0 \\ -1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

47

Another example

Assume for all neurons
r^{min}=10 Hz, r^{max}=100 Hz

r	50	70	10	30
	↓	→	↑	←
$\begin{bmatrix} x \\ y \end{bmatrix}$	$\begin{bmatrix} 0 \\ -1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

Linear algebra

- Left matrix: data
  - Rows: different data points
  - Columns: different features
- Right matrix: column contains weights for weighted sum

51

Matrices and weighted sums

r	1	4	1	0
	↓	→	↑	←
$\begin{bmatrix} x \\ y \end{bmatrix}$	$\begin{bmatrix} 0 \\ -1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$$1\begin{bmatrix} 0 \\ -1 \end{bmatrix} + 4\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0\begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

Matrix multiplication:
Sum {left row x right column}

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax + by + cz \\ dx + ey + fz \end{bmatrix}$$

52

Example Matrix multiplication

$$\begin{bmatrix}2 & 0 & -1\\ 3 & 1.5 & -2\end{bmatrix}\begin{bmatrix}0.5\\ -1\\ 2\end{bmatrix} = \begin{bmatrix}2\cdot 0.5 + 0\cdot(-1) - 1\cdot 2\\ 3\cdot 0.5 + 1.5\cdot(-1) - 2\cdot 2\end{bmatrix}$$

$$= \begin{bmatrix}1 + 0 - 2\\ 1.5 - 1.5 - 4\end{bmatrix} = \begin{bmatrix}-1\\ -4\end{bmatrix}$$

Alternatively:

$$\begin{bmatrix}2 & 0 & -1\\ 3 & 1.5 & -2\end{bmatrix}\begin{bmatrix}0.5\\ -1\\ 2\end{bmatrix} = 0.5\cdot\begin{bmatrix}2\\ 3\end{bmatrix} - 1\cdot\begin{bmatrix}0\\ 1.5\end{bmatrix} + 2\cdot\begin{bmatrix}-1\\ -2\end{bmatrix}$$

Linear algebra and distributed neural coding

Three features: [tired, happy, cold]

features -> brain response

[0.5, 0.1, 0.8] -> [49.1, 32.5, 22.8, 30.0, 36.4, 16.7]

[0.9, 0.4, 0.1] -> [33.2, 26.0, 15.1, 45.0, 36.8, 23.8]

[0.4, 0.9, 0.7] -> [45.9, 56.5, 27.7, 41.0, 63.6, 15.3]

[0.3, 0.1, 0.5] -> [30.5, 21.5, 14.5, 19.0, 24.0, 10.2]

Linear algebra and distributed neural coding

Three features: [tired, happy, cold]

features -> brain response

[0.5, 0.1, 0.8] -> [49.1, 32.5, 22.8, 30.0, 36.4, 16.7]

$$P\ F = R$$

$$\begin{bmatrix}| & | & |\\ patt1 & patt2 & patt3\\ | & | & |\end{bmatrix}\begin{bmatrix}0.5\\ 0.1\\ 0.8\end{bmatrix} =$$

$$0.5\cdot\begin{bmatrix}30\\ 10\\ \vdots\\ 20\\ 25\end{bmatrix} + 0.1\cdot\begin{bmatrix}5\\ 35\\ \vdots\\ 40\\ 2\end{bmatrix} + 0.8\cdot\begin{bmatrix}42\\ 30\\ \vdots\\ 28\\ 5\end{bmatrix} = \begin{bmatrix}49.1\\ 32.5\\ \vdots\\ 36.4\\ 16.7\end{bmatrix}$$

Linear algebra and distributed neural coding

Three features: [tired, happy, cold]

features -> brain response

[??, ??, ??] -> [30, 22, 15, 19, 24, 10]

$$P\ F = R$$

$$??\cdot\begin{bmatrix}30\\ 10\\ \vdots\\ 20\\ 25\end{bmatrix} + ??\cdot\begin{bmatrix}5\\ 35\\ \vdots\\ 40\\ 2\end{bmatrix} + ??\cdot\begin{bmatrix}42\\ 30\\ \vdots\\ 28\\ 5\end{bmatrix} = \begin{bmatrix}30\\ 22\\ \vdots\\ 24\\ 10\end{bmatrix}$$

$$P\ F = R$$