CISC 3250
Systems Neuroscience

Neural networks and information representation in computer science

March 3 edition

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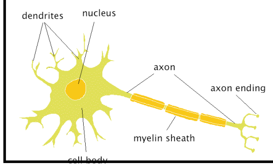
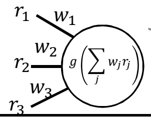
JMH 328A

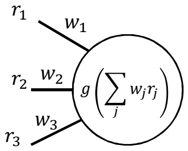


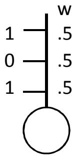


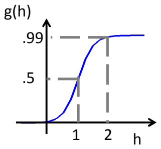
Artificial neuron – the perceptron

Perceptron – building block of artificial neural networks

- Weight inputs
- Perform summation
- Pass through non-linearity







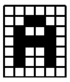
h=1x.5+0x.5+1x.5 = .5+.5 = 1

r_{out}=g(h)=g(1)=.5

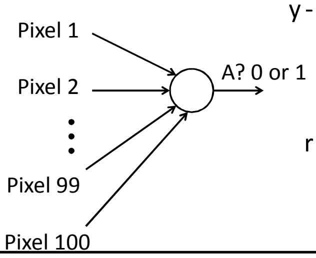
Example: Optical character recognition

Task is to identify a letter from a picture of that letter

x - input



Each pixel is 0 (white) or 1 (black)

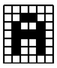


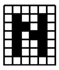
y - desired output

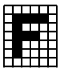
r - actual output

Feature vector input

x_k = [pixel 1, pixel 2, …, pixel 100]

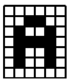
x_1 – pixels in 

x_{10} – pixels in 

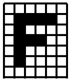
x_{15} – pixels in 

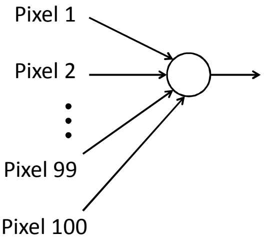
Learning to respond correctly

x_1



x_{15}





y_1=1

r_1=.3 Before learning

r_1=1 After learning

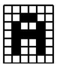
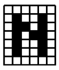
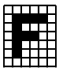
y_{15}=0

r_{15}=.7 Before learning

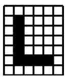
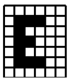
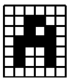
r_{15}=0 After learning

Training the perceptron

- Learn weights that will produced desired perceptron output for set of pictures of letters – **training set**

 y=1  y=0  y=0

- Evaluate these weights by testing correctness of perceptron output for separate set of pictures of letters – **testing set**

 y=0  y=0  y=1

Two learning approaches

Hebbian neurons: “cells that fire together, wire together”

Delta learning: Correcting weights to minimize error between perceptron output and expected output

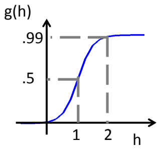
$$E = \frac{1}{2}\sum_i \left(r_i^{out} - y_i\right)^2$$

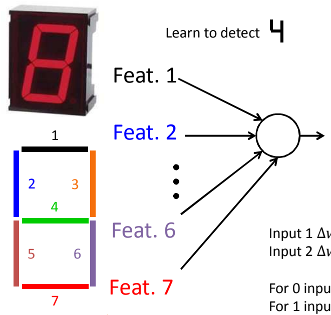
Perceptron math

Delta learning: Correcting weights to minimize error between perceptron output and expected output; *using weighted sum and sigmoid non-linearity g^{sig}*

$$\Delta w_{ij} = \overset{\text{Learning rate}}{\epsilon}\overbrace{\left(1 - r_i^{out}\right)}^{\text{Weight decay}}\underbrace{\left(\underset{\text{Desired output}}{y_i} - \underset{\text{Actual output}}{r_i^{out}}\right)}_{\text{Error correction}}\underset{\text{Hebb}}{\boxed{r_i^{out} r_j^{in}}}$$

Perceptron math example





w = [.2 .2 .2 .2 .2 .2 .2]
Input 4 , expect output y=1

h=0x.2+1x.2+1x.2+1x.2+0x.2+1x.2+0x.2=.8
r=g(.8)=.45

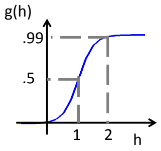
Input 1 Δw_{i1} = (1-.45) (1-.45) .45 0 = 0
Input 2 Δw_{i1} = (1-.45) (1-.45) .45 1 = .55 .55 .45 = .14

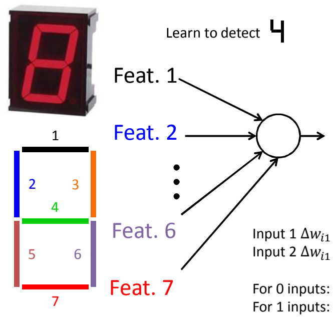
For 0 inputs: +0 -> .2+0 = .2
For 1 inputs: +.14 -> .2+.14=.34

Assume $\epsilon = 1$

$$\Delta w_{ij} = \epsilon\overbrace{\left(1 - r_i^{out}\right)}\overbrace{\left(y_i - r_i^{out}\right)}\overbrace{r_i^{out} r_j^{in}}$$

Perceptron math example





w^{new} = [.2 .34 .34 .34 .2 .34 .2]
Input 2 , expect output y=0

h=1x.2+0x.34+1x.34+1x.34+1x.2+0x.34+1x.2=1.28
r=g(.8)=.58

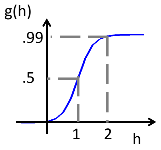
Input 1 Δw_{i1} = (1-.58) (0-.58) .58 1 = .42 -.42 .58 = -.10
Input 2 Δw_{i1} = (1-.58) (0-.58) .58 0 = 0

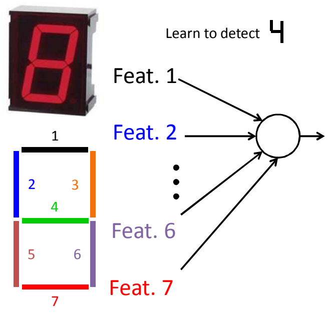
For 0 inputs: +0
For 1 inputs: -.1

Assume $\epsilon = 1$

$$\Delta w_{ij} = \epsilon\overbrace{\left(1 - r_i^{out}\right)}\overbrace{\left(y_i - r_i^{out}\right)}\overbrace{r_i^{out} r_j^{in}}$$

Perceptron math example



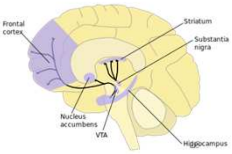


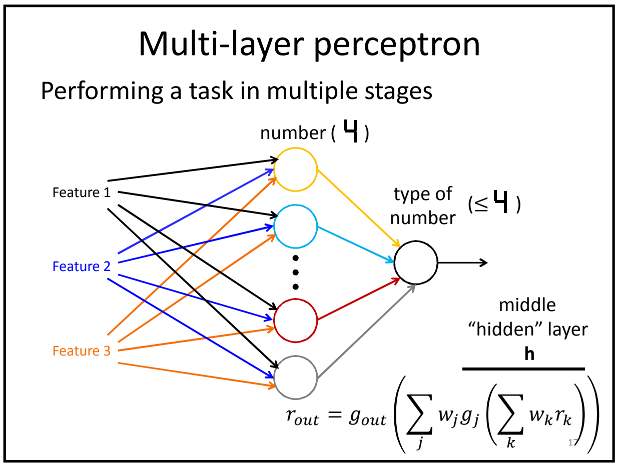
Eventually, we will get weights like:
w = [-2 .5 .5 .5 -2 .5 -2]

Requiring all edges of 4 to be present and no spurious edges

Hebb vs. delta learning

- Hebb is **unsupervised**
  - no “right answer” given
  - neuron/animal notices what inputs co-occur
- Delta rule is **supervised**
  - neuron instructed how to behave
  - mouse gets food reward for pushing lever -> mouse presses lever more often
  - in biology: dopamine reward feedback from ventral tegmental area (VTA) – in the limbic system

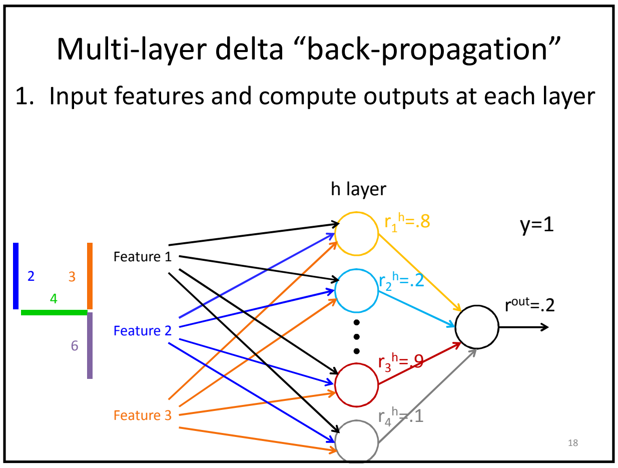




Multi-layer perceptron

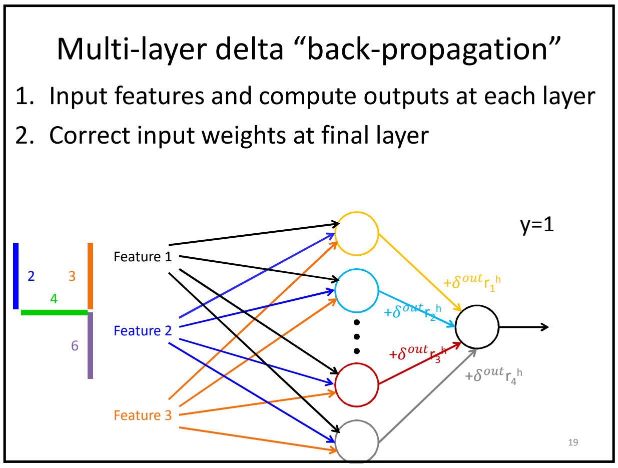
Performing a task in multiple stages

$$r_{out} = g_{out}\left(\sum_j w_j g_j\left(\sum_k w_k r_k\right)\right)$$



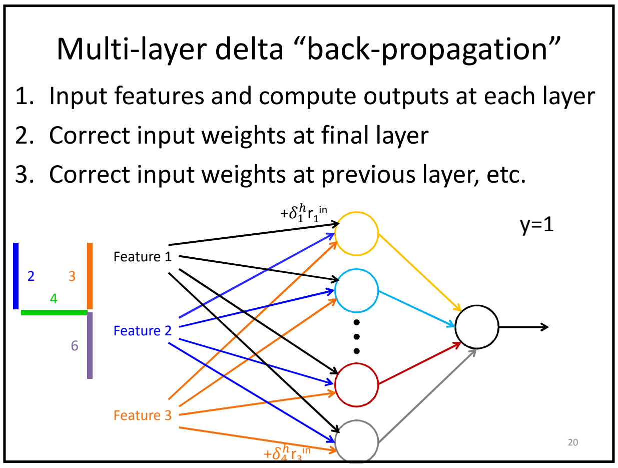
Multi-layer delta “back-propagation”

1. Input features and compute outputs at each layer



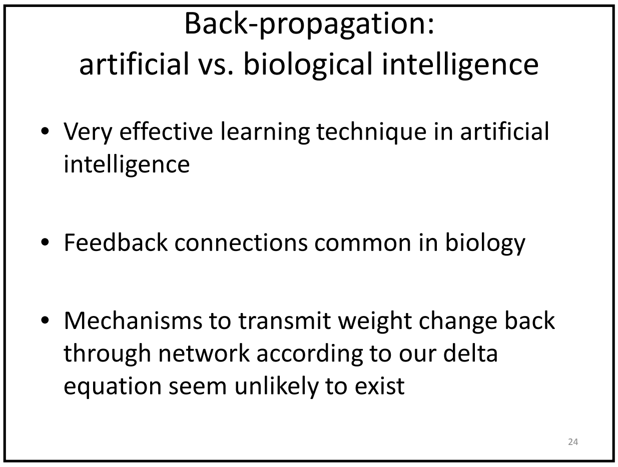
Multi-layer delta “back-propagation”

1. Input features and compute outputs at each layer
2. Correct input weights at final layer



Multi-layer delta “back-propagation”

1. Input features and compute outputs at each layer
2. Correct input weights at final layer
3. Correct input weights at previous layer, etc.



Back-propagation: artificial vs. biological intelligence

- Very effective learning technique in artificial intelligence
- Feedback connections common in biology
- Mechanisms to transmit weight change back through network according to our delta equation seem unlikely to exist

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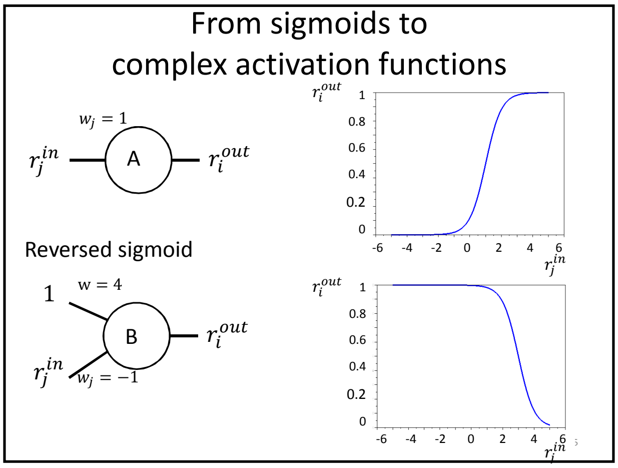
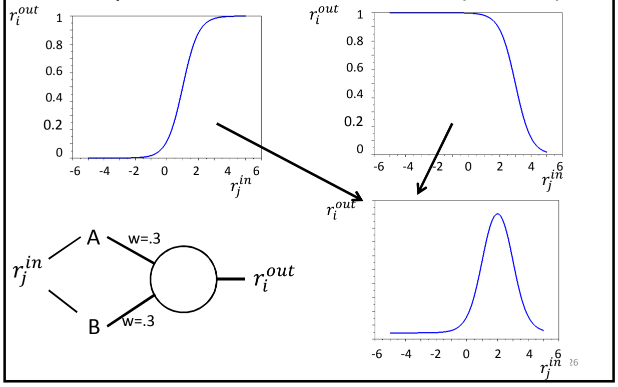


From sigmoids to complex activation functions

Reversed sigmoid

From sigmoids to complex activation functions (cont'd)



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Finding structure in data with perceptron learning

- If we can learn perceptron weights from a training set to predict correct outputs on testing set, there is a simple connection between the input features and the output
- Assign the input features to be variables in an experiment, assign 0-or-1 perceptron output to indicate condition under study

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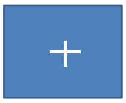
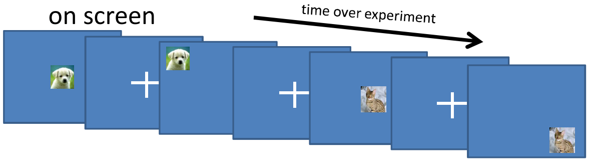
Example



- Have subject stare at center of screen
- Flash image of dog or cat very quickly (50 ms) on screen



- Ask subject to press button if they see a dog

Expect subjects will see dog only if dog appears at center of screen where subject is looking.

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Example

- x_{train} : [1 0 0 0 1 1 …], 1 if dog appears at center for a given display, 0 if dog at side
- y_{train} : [1 0 0 0 1 1 …], 1 if subject sees dog, 0 if subject does not see dog
- Learned perceptron weights will predict subject's future perception of dog based on picture location, because there is a connection between picture location and ability to rapidly perceived an image

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