Homework 0
(110 points total)
due September 9

Question 1: (16 points) Examine the following “set-builder” descriptions of the following sets, and provide a list of set members.

**Example “builder” description:** \{y \mid y=3x \text{ and } x \in \mathbb{N}\}

**Example answer:** \{3, 6, 9, 12, \ldots \}

a. \{(y-3, y) \mid y \in \mathbb{N}\}
b. \{8m \mid m \in \mathbb{N} \text{ and } m<5\}
c. \{\frac{x^2}{3} \mid x \in \mathbb{Z}\}
d. \{w \mid w \text{ is a string of As and Bs, and the number of As in the string is the same as the number of Bs in the string}\}

Question 2: (20 points) Provide a “set-builder” description (see Question 1) for each of the sets with elements listed below.

a. \{
\}
b. \{\ldots, -8, -6, -4, -2\}
c. \{10, 100, 1000, 10000, \ldots\}
d. \{3, 6, 12, 24, 48, \ldots\}
e. \{\ldots, -10, -8, -6, -4, -2\}

Question 3: (28 points) Let $A=\{ab, abab, ababab, abababab, ababababab\}$, $B=\{ab, abab, ababab\}$, and $C=\{ab, aabb\}$

a. True or False: $C \subseteq A$
b. True or False: $A \subseteq B$
c. What is $B - C$?
d. What is $A \cap B$?
e. What is the power set of $C$?

For 3f and 3g, explain your answer **without** explicitly listing and counting all the member of the power and Cartesian-product sets.
f. How many elements are in the power set of $B$?
g. How many elements are in the “Cartesian product” (also known as the “set product”) $(A \cup B) \times C$?
Question 4: (14 points) Let $X$ be the set $\{2, 4, 6, 8, 10\}$ and $Y$ be the set $\{1, 2, 3, 4, 5\}$. The unary function $f: X \rightarrow Y$ and the binary function $g:X \times Y \rightarrow X$ are described in the following tables.

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<tr>
<th>$n$</th>
<th>$f(n)$</th>
<th>$g$</th>
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a. What is the value of $f(8)$?
b. What are the range and domain of $f$?
c. What is the value of $g(8, 4)$?
d. What is the value of $f(g(6, 5))$?

Question 5: (12 points) Consider the undirected graph $G=(V,E)$ where $V$, the set of nodes, is $\{1, 2, 3, 4\}$ and $E$, the set of edges, is $\{\{1,4\}, \{2,3\}, \{2,4\}, \{3,1\}\}$.

a. Draw the graph $G$.
b. What are the degrees of each node?
c. Write a set of edges forming a path from node 3 to node 4 in the graph.

Question 6: (10 points) Write a formal description of the following graph.

![Graph](image)

Question 7: (10 points) Show that every graph with two or more nodes contains at least two nodes that have equal degrees. (Note, we do not allow an edge from a node to itself.)

*Hint: Think of the pigeonhole principle (taught in CISC 1100/1400)!*