1. Consider the state diagrams for two DFAs, M1 and M2

- **M1**: \( q_1 \rightarrow 0, q_2 \rightarrow 1, q_3 \rightarrow 0,1 \)
- **M2**: \( q_1 \rightarrow 1, q_2 \rightarrow 0, q_3 \rightarrow 0,1 \)

### a. Formal descriptions

For each machine, the elements of the 5-tuple \((Q, \Sigma, \delta, q_0, F)\) are as follows:

- **M1**:
  - \( Q = \{q_1, q_2, q_3\} \)
  - \( \Sigma = \{0, 1\} \)
  - \( \delta = \delta_{M1} \)
  - \( q_0 = q_1 \)
  - \( F = \{q_2\} \)

- **M2**:
  - \( Q = \{q_1, q_2, q_3\} \)
  - \( \Sigma = \{0, 1\} \)
  - \( \delta = \delta_{M2} \)
  - \( q_0 = q_1 \)
  - \( F = \{q_3\} \)

### b. Sequence of states

- **M1**: \( q_1, q_2, q_3, q_1 \)
- **M2**: \( q_2, q_3, q_3, q_3 \)

### c. Acceptance of \( \varepsilon \)

- **M1**: no
- **M2**: no

### d. Acceptance of string 110011

- **M1**: no
- **M2**: yes

2. The formal description of a DFA M3 is \( (Q, \Sigma, \delta, q_0, F) = (\{q_1, q_2, q_3, q_4\}, \{0, 1\}, \delta_{M3}, q_1, \{q_3\}) \) where \( \delta \) is provided by the following table:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>q1</td>
<td>q3</td>
<td>q4</td>
</tr>
<tr>
<td>q2</td>
<td>q1</td>
<td>q2</td>
</tr>
<tr>
<td>q3</td>
<td>q1</td>
<td>q3</td>
</tr>
<tr>
<td>q4</td>
<td>q2</td>
<td>q4</td>
</tr>
</tbody>
</table>

Give the state diagram for this machine.
3. Give state diagrams of DFAs recognizing the following languages. In all parts, $\Sigma = \{0,1\}$.

a. \{w \mid \text{the number of digits in } w \text{ is a multiple of four} \} 

b. \{w \mid \text{the digits interpreted in binary are evaluated to 16 or higher} \} 

c. The empty set 

d. \{w \mid \text{w has fewer than 2 digits OR the fifth digit is 0} \}
e. \{w \mid \text{the digits together sum to the number 2}\}

4. Let $A_k=\{w \mid w \text{ has a 1 at the } k^{th} \text{ position in the string}\}$. For example, $A_2$ includes 0100, 1110, 010, 11; $A_4$ includes 00010, 1111, 0111, 110100.

Show that for each $k>1$, the language $A_k$ is regular.

If language is regular, can make NFA of DFA to recognize it.

We can make NFA to recognize any $A_k$ using this approach:

$N_k = (\{q_0, \ldots, q_k\}, \{0,1\}, \delta_k, q_0, \{q_k\})$

$\delta_k(q_i, a) = \{q_{i+1}\}$ if $i<k-1$ and $a \in \Sigma$

$\delta_k(q_{k-1}, 0) = \{q_k\}$

$\delta_k(q_{k-1}, 1) = \{\}$

5. Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts, $\Sigma = \{a,b\}$

a. Accepts $\{w \mid w \text{ has exactly 2 a's and an odd number of b's}\}$ using SIX states.

(Example accept strings: abbab, baa)

b. Accepts $\{w \mid w \text{ ends in abaa}\}$ using FIVE states.

(Example accept strings: babaa, abaaabaa)

6. Use Theorem 1.45 in the text (also given in class) to provide an NFA state diagram for a machine that recognize the union of the languages $L_1$ and $L_2$ below. Note $\Sigma = \{0,1\}$.

$L_1 = \{w \mid \text{every even position in } w \text{ has 1}\}$

$L_2 = \{w \mid w \text{ has 2 or more characters, and the first symbol is 1}\}$
7. Use Theorem 1.47 in the text (also given in class) to provide an NFA state diagram for a machine that recognizes the concatenation of the languages $L_1$ and $L_2$ from question 6.

![NFA Diagram]

8. 
   a. Show that if $M$ is a DFA that recognizes language $B$, swapping the accept and nonaccept states in $M$ yields a new DFA recognizing the complement of $B$. Conclude that the class of regular languages is closed under complement.

   The complement of $B$ ($B'$) is the language containing every string not in $B$. Each string not in $B$ will bring you to a state in $M$ that is NOT an accept state. In the machine $M'$, each of these states ARE accept states. Furthermore, every string in $B$ will bring you to an accept state in $M$; and will bring you to a NON-accept state in $M'$. Thus, $M'$ recognizes $B'$ – it accepts every string in $B'$. Thus $B'$ is regular, since there exists a DFA that recognizes it. Therefore, regular languages are closed under complement.

   b. Show by giving an example that if $M$ is an NFA that recognizes language $C$, swapping the accept and nonaccept states in $M$ doesn’t necessarily yield a new NFA that recognizes the complement of $C$. Is the class of languages recognized by NFAs closed under complement? Explain your answer.

   ![NFA Diagram]

   The string $y$ is rejected by this NFA and bye the complement of this NFA

   The class of languages recognized by NFAs is closed under complement because every language recognized by an NFA is recognized by a DFA, so it is regular. The complement of each regular language is also regular, which means an NFA can be defined to recognize it.
9. Convert the following NFA to a DFA. You may wish to use the lecture/class method used to prove NFA-DFA equivalence.