

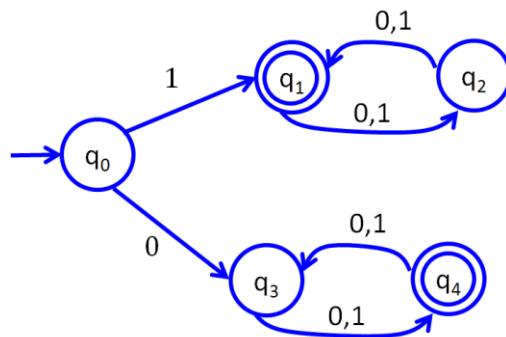
1. Consider the state diagram for the following DFAs. For each, answer the following questions:

(1) What state is reached by the input:  $w=00110$  ?

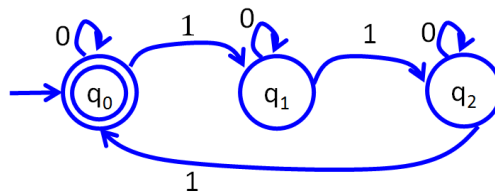
(2) What is the transition function?

(3) What is the language recognized?

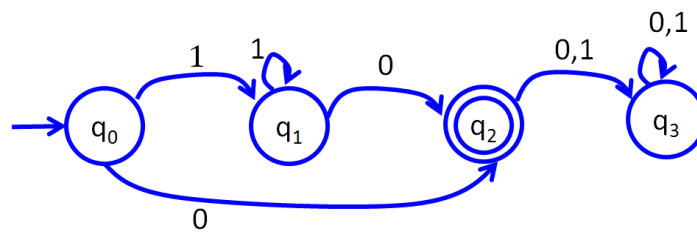
**M1:**



**M2:**



**M3:**



2. Define a machine to recognize the following languages in the alphabet

$$\Sigma = \{1,2,3\}$$

(5 points)

**L4={w | the product of input symbols is even}** E.g., 111  $\rightarrow 1 \times 1 \times 1 = 1$  is odd-reject,  
233  $\rightarrow 2 \times 3 \times 3 = 18$  is even-accept

**L5={w | numbers entered in non-decreasing order}** Examples: 112223, 122333

**L6={w | first two symbols are identical}** Examples: 001213, 333212, 3310013

3. Prove the following languages are regular, using the alphabet  $\Sigma = \{a, b, c\}$ :

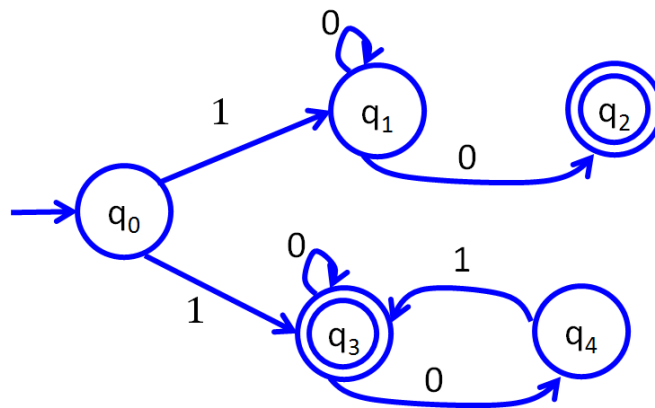
**L7={w | w contains an odd number of b's}**

**L8={w | w contains the sequence bcb}** (Examples: aabbbcb or cbcba)

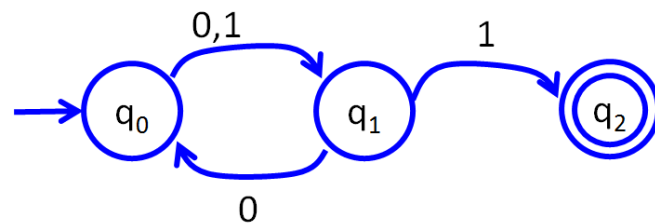
**L9={w | w does not have three a's in a row}**

4. Consider the following NFAs. For each, answer:
- (1) what state(s) will be reached by the input: 0011
  - (2) provide a regular expression to describe the recognized language
  - (3) For **N11** and **N12**, convert NFA to DFA

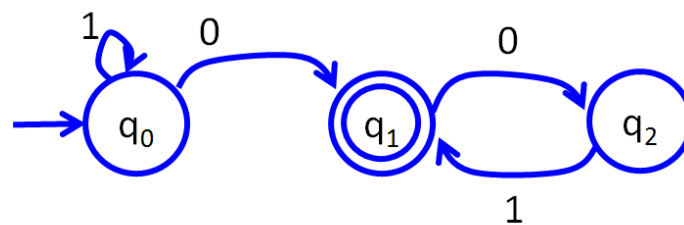
**N10:**



**N11:**



**N12:**



5. For each regular expression using  $\Sigma = \{a, b\}$ :

(1) Provide three example words.

(2) Convert these regular expressions to a DFA or NFA

**L13={ab\*(ba)\*}**

**L14={(a ∪ b)ba\*}**

**L15={(bb)\* ∪ (aa)\*}**

6. What is the minimum pumping length for each of these languages, showing these languages are regular? We use the alphabet  $\Sigma = \{0,1,2\}$

**L16={00(0 ∪ 1)\*12}**

**L17={0(22)\*10}**

**L18={111(202)\*210}**

If pumping length is  $p=5$ , how would you break up string  $w$  into  $x$ ,  $y$ , and  $z$  for languages  $L$  below?

**L19={ 20(11)\*001 }, w=201111001**

**L20={ (121)\*001 } w=121001**

7. Consider the language  $L_{21} = \{01(101)^*11\}$ , what is the error in each of the following “pumping lemma” arguments?

**Argument 1:** Let us take  $w=0111$ ,  $w \in L_{21}$ . We cannot divide  $w=xyz$  such that  $y^i z \in L_{21}$ ,  $i \geq 0$ . For example, if  $x=0$ ,  $y=11$ , and  $z=1$ ,  $xy^2z = 011111 \notin L_{21}$ . Therefore,  $L_{21}$  is not regular.

**Argument 2:** Let us take  $w=0110110111$ ,  $w \in L_{21}$ . If we divide  $w=xyz$  as follows:  $x=0110110$ ,  $y=11$ ,  $z=1$ , we cannot repeat  $y$  such that  $xy^i z \in L_{21}$ ,  $i \geq 0$ . For example, if  $xy^2z = 011011011111 \notin L_{21}$ . Therefore,  $L_{21}$  is not regular.

8. Prove these languages are not regular.

$L_{24} = \{0^n 1^{2n} 0^{3n} \mid n > 0\}$

$L_{25} = \{1^{n^3} \mid n > 0\}$

9. For each of the following grammars, list three strings produced by the grammar

**G26:**

$S \rightarrow AB \mid BA$

$A \rightarrow xAy \mid \epsilon$

$B \rightarrow BzB \mid y$

**G27:**

$S \rightarrow A \mid AA$

$A \rightarrow 00 \mid 11$

**G28:**

$A \rightarrow 11A00 \mid \epsilon$

10. Provide the languages described by two of the grammars:

**G27 (from above)**

**G28 (from above)**

10. Provide a grammar to produce the following languages

**L32 =  $\{0^n(11)^n \mid n \geq 0\}$**

**L33 =  $\{01^*00^*\}$**

**L34 =  $\{w \mid w = w^{\text{Reverse}}\}$     Examples: 00100, 10101, 1111**

11. Convert the following grammars to Chomsky Normal Form

**G29:**

$S \rightarrow xAy \mid BA$

$A \rightarrow z \mid AzA$

$B \rightarrow yB \mid \varepsilon$

**G30:**

$S \rightarrow BAB \mid ABA$

$A \rightarrow y \mid z$

$B \rightarrow x \mid AA \mid \varepsilon$

**G31:**

$S \rightarrow ByBy$

$B \rightarrow xBx \mid \varepsilon$