1. Provide two valid strings in the languages described by each of the following regular expressions, with alphabet $\Sigma = \{0,1,2\}$.

(a) $0(010)^*1$

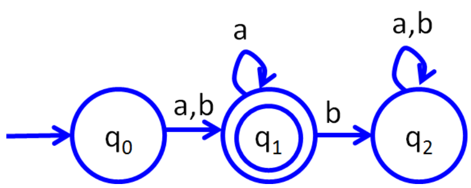
(b) $(21 \cup 10)^*0012^*$

Examples: 001, 001222, 21001, 10001, 210012, 2121001222, 102121001

(c) $1^*(200)^* \cup 100^*01$

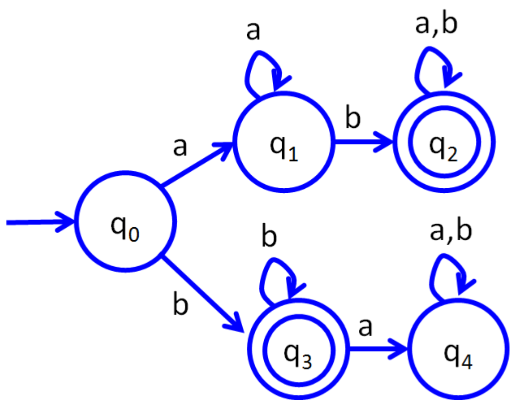
2. For each of the following DFAs, provide a Regular Expression to describe the language, with alphabet $\Sigma = \{a, b\}$.

(a) RED QUESTION

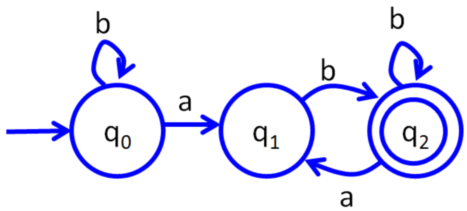


(aUb)a*

(b) BLUE QUESTION



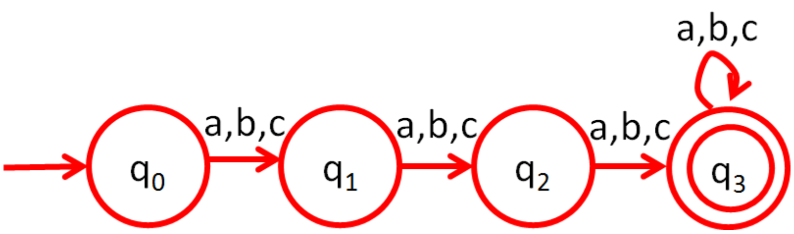
(c) GREEN QUESTION



3. Create a DFA to accept each of the following languages.

A={w | last number in w is even} , given alphabet $\Sigma = \{0,1,2,3\}$

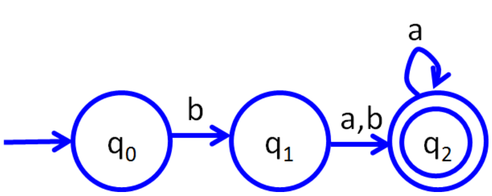
B={w | at least three symbols in w} , given alphabet $\Sigma = \{a, b, c\}$

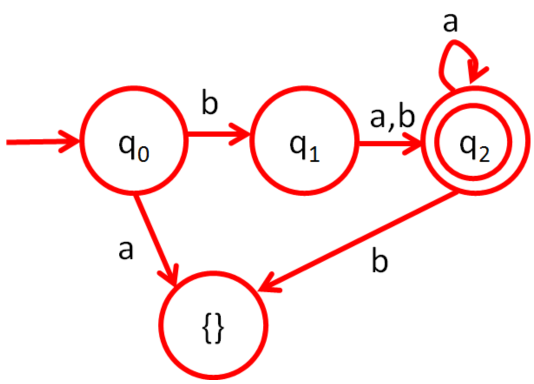


C={w | sum of digits in w equals 2} , given alphabet $\Sigma = \{0,1,2\}$

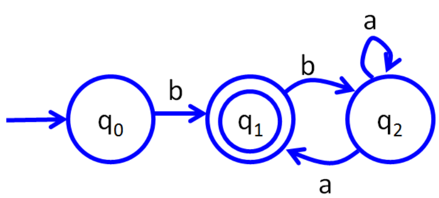
4. Convert each of the following NFAs to a DFA, with alphabet $\Sigma = \{a, b\}$.

(a) RED QUESTION

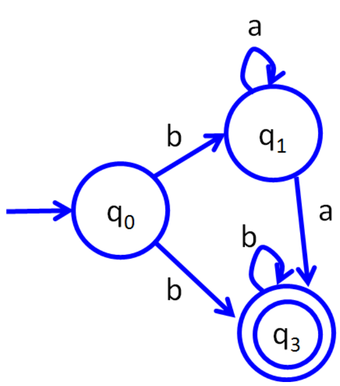




(b) GREEN QUESTION



(c) BLUE QUESTION



5. Prove the following languages are **not** regular.

(a) $A=\{b^{k!}a \mid k>0\}$

(b) $B=\{0^k1^{2k}0^k \mid k>0\}$
Pumping lemma!
$w = 0^p1^{2p}0^p$ $x=0^m$ $y=0^n$ $z=0^{p-(m+n)}1^{2p}0^p$ $p>=n>0$
If $w \in B$, then must be $xy^2z \in B$
$xy^2z = 0^{p+n}1^{2k}0^p$ First number of 0's now is **not** half the number of 1's, so xy^2z is NOT in language B. This means w was not pumpable and B is not regular!

7. Provide two valid strings for each of the following CFGs.

(a) G1:

S -> A | B
A -> DC | C
B -> EF | F
C -> dog | cat | mouse
D -> big | small | red | white
E -> quickly | slowly
F -> runs | swims | jumps | barks

(b) G2:

S -> BA | B
B -> xBx | ε
A -> c | de | f

B -> **ε**
B -> xBx -> xxεxx -> **xxxx**
BA -> εde -> **de**
BA -> xεxc -> **xxc**

(c) G3:

S -> CaC | C
C -> yCy | y

8. Convert the following CFGs to CNF (same as Q7).

(a) G1: (for G1, each word is a terminal)

S -> A | B
A -> DC | C
B -> EF | F
C -> dog | cat | mouse
D -> big | small | red | white
E -> quickly | slowly
F -> runs | swims | jumps | barks

(b) G2:

S -> BA | B
B -> xBx | ε
A -> c | de | f

(c) G3:

S -> CaC | C
C -> yBy | y

S -> CaC | yBy | y *replace C*
C -> yBy | y

S -> CAC | YBY | y *replace literals with variables*
A -> a
Y -> y
C -> YBY | y

S -> CD | YE | y *replace 3-variable terms with 2-var terms*
D -> AC
E -> BY
A -> a
Y -> y
C -> YE | y

9. Express each of the following languages as a **CFG**.

(a) A = $\{x^k y^{2k} z\}$

S -> Bz

B -> xByy | ε

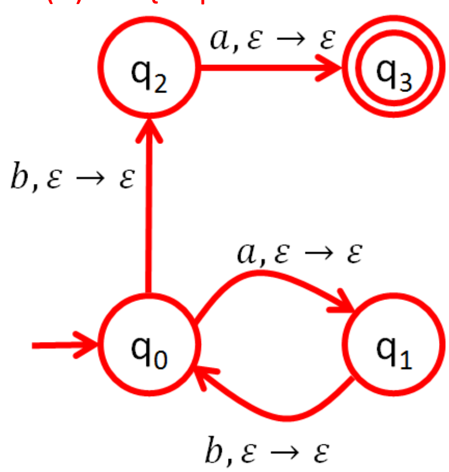
(b) B = {w | w is described by (ab)*ba }

(c) C = $\{ 010^k 101^{k+2} \mid k > 0 \}$

10. Describe the PDA to accept each of the following languages (languages from Q9).

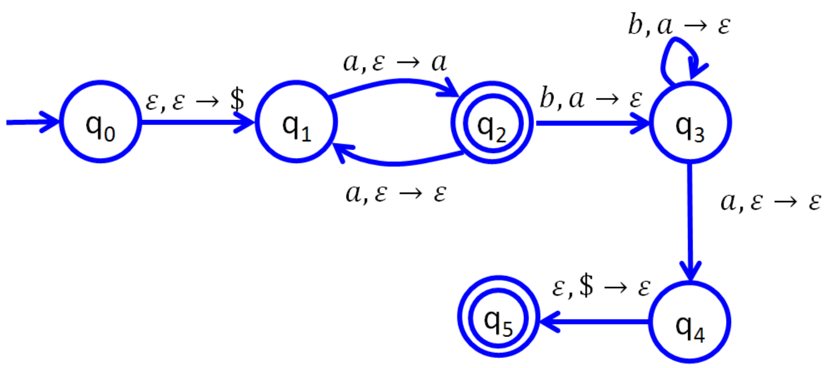
(a) A = $\{x^k y^{2k} z\}$

(b) B = {w | w is described by (ab)*ba }



(c) C = $\{ 010^k 101^{k+2} \mid k > 0 \}$

11. What is the response of PDA P1 to each input: **i.e., does it reach an accept state?**



Input 1: bbaa

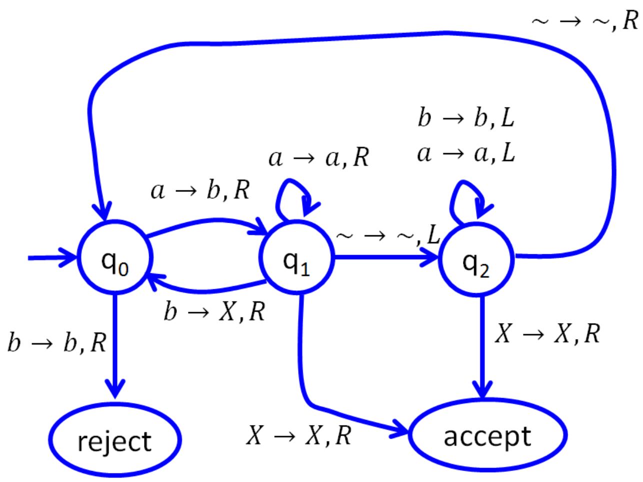
Does not reach accept state! (It starts with b, so quickly departs NFA.)

Input 2: aaa

Input 3: abb

Input 4: aaaaabbba

12. Describe the configurations resulting from each of the input tapes specified below for the following Turing Machine.



(a) aabb
q_0 aabb
q_1 babb
q_1 babb
q_0 baXb
reject baXb~

(b) abaaa

(c) aaaba

13. Express the following problems as languages.

(a) Determine if two specified CFG's accept complementary inputs – every accepted input for the first CFG is rejected by the second CFG and vice versa.

(b) Determine if a specified DFA accepts a specified string repeated zero or more times.

(c) Determine if a specified Turing machine accepts the same language as a specified PDA.
L = {<P,T> | L(P) = L(T)}

14. Prove the follow languages are decidable.

(a) Determine if a specified DFA accepts a specified string repeated zero or more times.

(b) Determine if a specified CFG is in Chomsky Normal Form.

(c) Determine if a specified CFG does not accept a specified word.
Generate all words of length |w|. If one of these words is the originally specified word, reject. Otherwise accept.

15. Provide a big-O and a little-o complexity for each function.

(a) $f(n) = 20\ n \log n + 5n + 2$
Smallest: $O(n \log n)$ Also: $O(n^2)$, $O(n^3)$, $O(2^n)$
Near-smallest: $o(n^2)$, $o(n \log^2 n)$; also: $o(2^n)$, $o(n^6)$

(b) $f(n) = 30\ n^3 + 6\ n^5 + \log n$

(c) $f(n) = 5 n^2 + n^3 \log n + 4^n + 8$

16. Compute the complexity for each algorithm described below.

(a) Algorithm 1: (State the complexity based on *r* and *c*)
Start with a table of *r* rows and *c* columns

1. Sum the elements in each row
   - Use a running sum with a loop across all columns
2. Find the row with the maximum sum
   - Loop through all rows, saving biggest sum and its row in two separate variables

(b) Algorithm 2: (State the complexity based on *n*)
Start with a list of *n* elements

1. While list is longer than 1 element long
   - Replace each pair of elements with the product of the two elements (elements 1 and 2 replaced by single product, elements 3 and 4 replaced by single product, elements 5 and 6 replaced by single product, etc.)

Number of loop repeat: $\log_2 n$; time to compute products: O(n/2)=O(n)
In total: **O(n log n)**

17. Determine if the following problems are in P and/or NP.

(a) Given a directed graph and two nodes a and b, determine if there are at least two different paths to get from node a to node b. Paths are "different" if they differ by at least one edge.

(b) In an undirected graph, determine if every node is attached to every other node.

This is effectively finding a clique of size n where n is the number of nodes. However, you only need to test **ONE** clique – the one containing ALL nodes. Testing one solution takes polynomial time. **So this problem actually is in P (and also in NP since all P problems are also in NP).**

(c) Determine if the language of a DFA is empty.