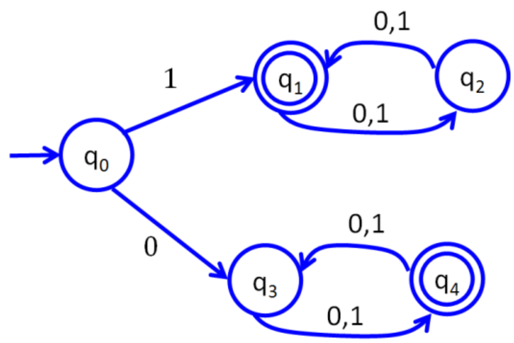
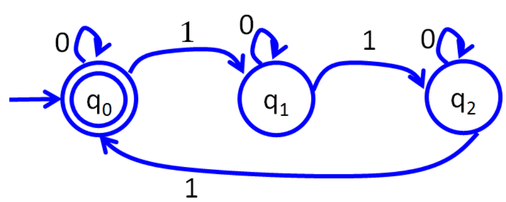
1. Consider the state diagram for the following DFAs. For each, answer the following questions:
(1) What state is reached by the input: w=00110 ?
(2) What is the transition function?
(3) What is the language recognized?

M1:



M2:



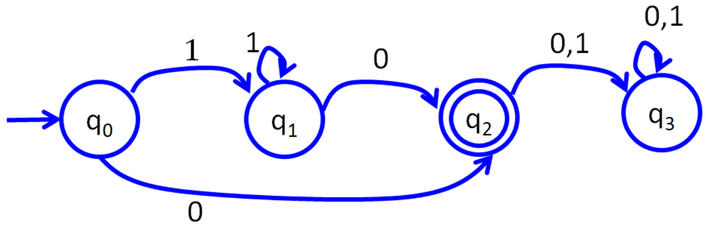
(1) **q_2** q_0 (0) -> q_0 0 -> q_0 (1) -> q_1 (1) -> q_2 (0) -> q_2

(2)

	0	**1**
q_0	**q_0**	**q_1**
q_1	**q_1**	**q_2**
q_2	**q_2**	**q_0**

(3) **{w | the number of 1's entered is a multiple of 3}**
$\{0^*(10^*10^*10^*)^*\}$

M3:

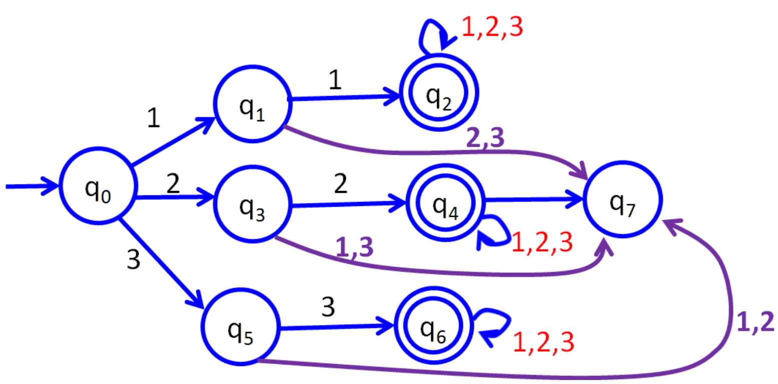


2. Define a machine to recognize the following languages in the alphabet $\Sigma = \{1,2,3\}$
(5 points)

L4={w | the product of input symbols is even} E.g., 111->1x1x1=1 is odd-reject, 233 -> 2x3x3 = 18 is even-accept

L5={w | numbers entered in non-decreasing order} Examples: 112223, 122333

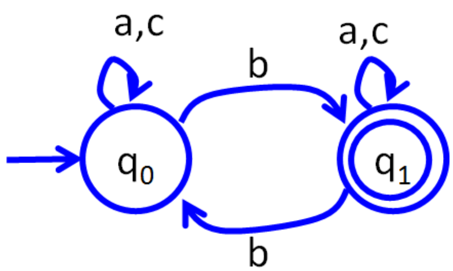
L6={w | first two symbols are identical} Examples: 001213, 333212, 3310013



3. Prove the following languages are regular, using the alphabet $\Sigma = \{a, b, c\}$:

L7={w | w contains an odd number of b's}

Define a DFA to detect the language and/or show a regular expression captures the language.



$(aUc)^*b(aUc)^*((aUc)^*b(aUc)^*b(aUc)^*)$

L8={w | w contains the sequence bcb} (Examples: aabbbcbb or ccbcba)

L9={w | w does not have three a's in a row}

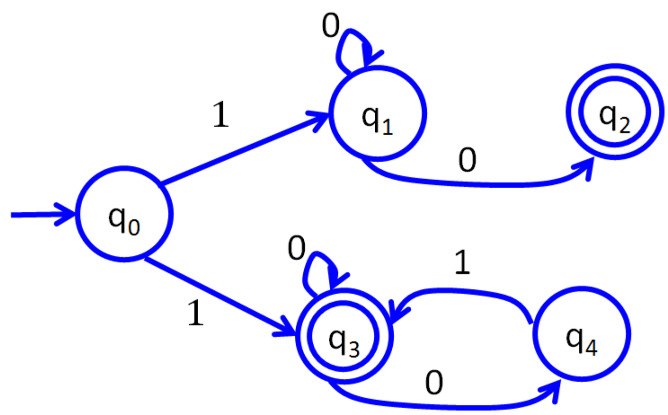
4. Consider the following NFAs. For each, answer:

(1) what state(s) will be reached by the input: 0011

(2) provide a regular expression to describe the recognized language

(3) **For N11 and N12**, convert NFA to DFA

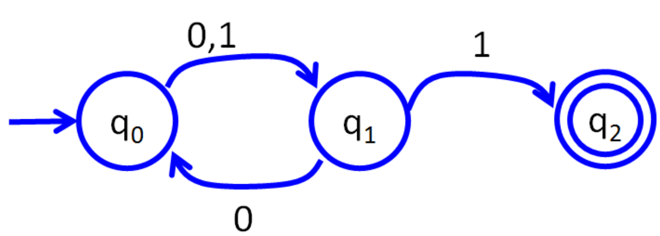
N10:



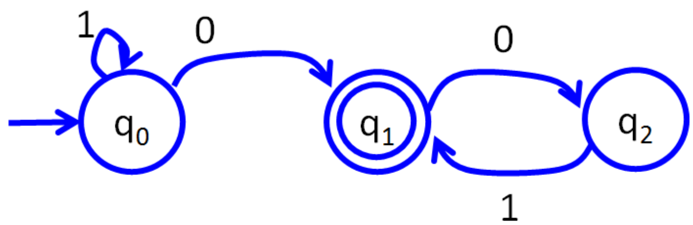
(1) No state – it will be rejected!

(2) **$1(0^{*}\ U\ 0^{*}\ (0^{*}01)^{*}$**

N11:



N12:

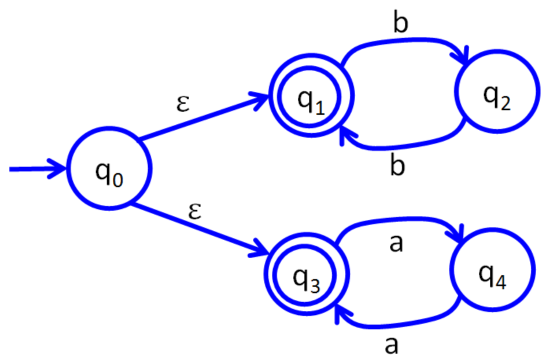


5. For each regular expression using $\Sigma = \{a, b\}$:
(1) Provide three example words.
(2) Convert these regular expressions to a DFA or NFA

L13=$\{ab^*(ba)^*\}$

L14=$\{(a \cup b)ba^*\}$

L15=$\{(bb)^* \cup (aa)^*\}$
(1) Examples: **bb, aa, bbbbbb, aaaaaa, ε**
(2)



6. What is the minimum pumping length for each of these languages, showing these languages are regular? We use the alphabet $\Sigma = \{0,1,2\}$

L16=$\{00(0 \cup 1)^*12\}$

L17=$\{0(22)^*10\}$
p=5, minimum pumpable string is 02210

L18=$\{111(202)^*210\}$

If pumping length is p=5, how would you break up string w into x, y, and z for languages L below?

L19={ 20(11)*001 }, w=201111001

L20={ (121)*001 } w=121001

7. Consider the language L21 ={01(101)*11}, what is the error in each of the following "pumping lemma" arguments?

Argument 1: Let us take w=0111, $w \in L21$. We cannot divide w=xyz such that $y^i z \in L21, i \geq 0$. For example, if x=0, y=11, and z=1, xy^{2}z = 011111$\notin L21$. Therefore, L21 is not regular.

The pumping length is p=7. Using any strings in L21 with length less than pumping length is not necessarily pumpable, and the inability to pump a too-short string does not prove anything. You can only test pumping on strings with at least as many characters as the pumping length.

Argument 2: Let us take w=0110110111, $w \in L21$. If we divide w=xyz as follows: x=0110110 , y=11, z=1, we cannot repeat y such that $xy^i z \in L21, i \geq 0$. For example, if xy^{2}z = 011011011111$\notin L21$. Therefore, L21 is not regular.

8. Prove these languages are not regular.

$L24=\{0^n1^{2n}0^{3n} \mid n>0\}$
Proof by contradiction with pumping lemma:
Assume L24 is pumpable. Now consider $w=0^p2^{2p}0^{3p}$, which is element of L24 with $|w|>p$. Thus, w must be pumpable.
w=xyz $x=0^j$ $y=0^k$ $z=0^{(p-(j+k))}2^{2p}0^{3p}$ $j+k \leq p$
Try pumping w: xy^2z -> $0^j\ 0^{2k}\ 0^{p-(j+k)}2^{2p}0^{3p}$
xy^2z begins with j+2k+p-(j+k) 0's … j+2k+p-(j+k) = *p+k* 0's
xy^2z begins with p+k 0's followed by 2p 1's.
$2p \neq p+k$, so $xy^2z \notin L24$, which means L24 is not regular!

$L25=\{1^{n^3} \mid n>0\}$

9. For each of the following grammars, list three strings produced by the grammar

G26:
S -> AB | BA
A -> xAy | ε
B -> BzB | y

Examples: AB -> εy -> **y**
BA -> yxAy -> yxxAyy -> yxxεyy -> **yxxyy**
AB -> εBzB -> εBzBzy -> εBzBzyzy -> **εyzyzyzy**

G27:
S -> A | AA
A -> 00 | 11

G28:
A -> 11A00 | ε

10. Provide the languages described by two of the grammars:

G27 (from above)

G28 (from above)

10. Provide a grammar to produce the following languages

L32 = $\{0^n(11)^n \mid n \geq 0\}$

L33 = $\{01^*00^*\}$

L34 = $\{w \mid w=w^{Reverse}\}$ Examples: 00100, 10101, 1111
S -> 0S0 | 1S1 | 0 | 1 | ε

11. Convert the following grammars to Chomsky Normal Form

G29:
S -> xAy | BA
A -> z | AzA

B -> yB | ε

Remove the ε

S -> xAy | BA | A

A -> z | AzA

B -> yB | y

Remove the S->A unit rule

S -> xAy | BA | z | AzA

A -> z | AzA

B -> yB | y

Replace mixed terminal—variable rules with variables-only rules

S -> U_xAU_y | BA | z | AU_zA

U_x -> x

U_y -> y

U_z -> z

A -> z | AU_zA

B -> U_yB | y

Replace 3$^+$ variable rules with 2-variables rules

S -> U_xC | BA | z | AD

C -> AU_y

D -> U_zA

U_x -> x

U_y -> y

U_z -> z

A -> z | AD

B -> U_yB | y

G30:

S -> BAB | ABA

A -> y | z

B -> x | AA | ε

G31:

S-> ByBy

B -> xBx | ε