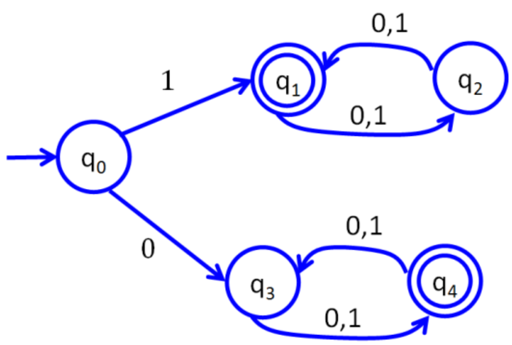
f1. Consider the state diagram for the following DFAs. For each, answer the following questions:

(1) What state is reached by the input: w=00110 ?

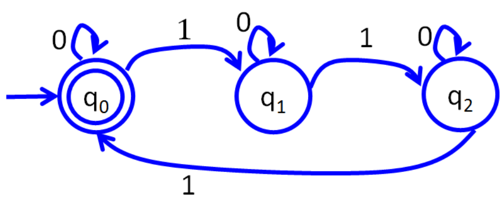
(2) What is the transition function?

(3) What is the language recognized?

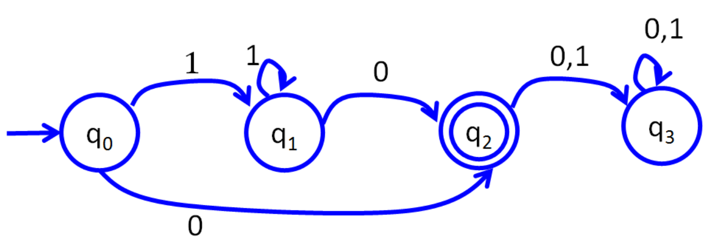
M1:



M2:



M3:



(1) **q_3** q_0 (0) -> q_2 (0) -> q_3 (1) -> q_3 (1) -> q_3 (0) -> q_3

(2)

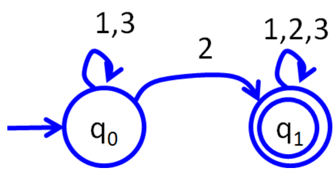
	0	**1**
q_0	**q_2**	**q_1**
q_1	**q_2**	**q_1**
q_2	**q_3**	**q_3**
q_3	**q_3**	**q_3**

(3) **L3 = {w | contains exactly one 0, followed by no other symbols} 1^*0**

2. Define a machine to recognize the following languages in the alphabet
$\Sigma = \{1,2,3\}$
(5 points)

L4={w | the product of input symbols is even} E.g., 111->1x1x1=1 is odd-reject, 233 -> 2x3x3 = 18 is even-accept

Presuming initial running product is 1:



$(1U3)^*2(1U2U3)^*$

L5={w | numbers entered in non-decreasing order} Examples: 112223, 122333

L6={w | first two symbols are identical} Examples: 001213, 333212, 3310013

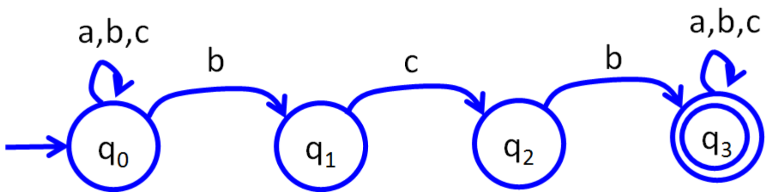
3. Prove the following languages are regular, using the alphabet $\Sigma = \{a, b, c\}$:

L7={w | w contains an odd number of b's}

L8={w | w contains the sequence bcb} (Examples: aabbbcbb or ccbcba)

Prove this by showing an FSA that recognizes L8 or showing regular expression for L8

NFA:

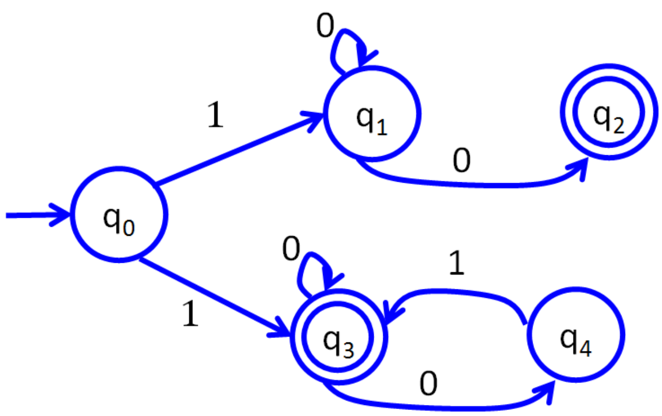


RegExp: **$(aUbUc)^*bcb(aUbUc)^*$** or $\Sigma^*bcb\Sigma^*$

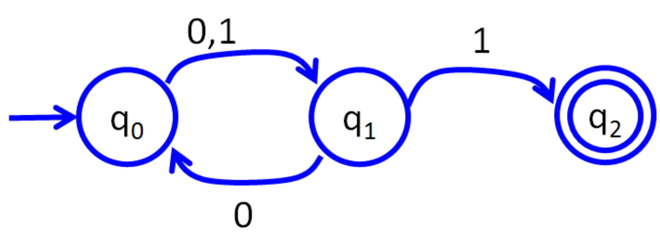
L9={w | w does not have three a's in a row}

4. Consider the following NFAs. For each, answer:
(1) what state(s) will be reached by the input: 0011
(2) provide a regular expression to describe the recognized language
(3) **For N11 and N12**, convert NFA to DFA

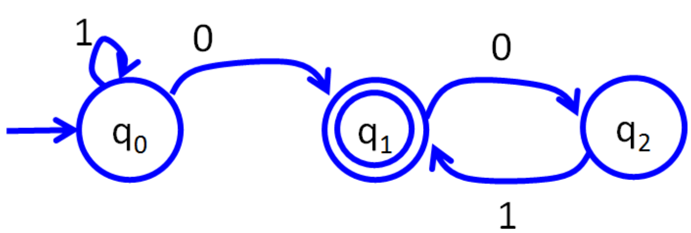
N10:



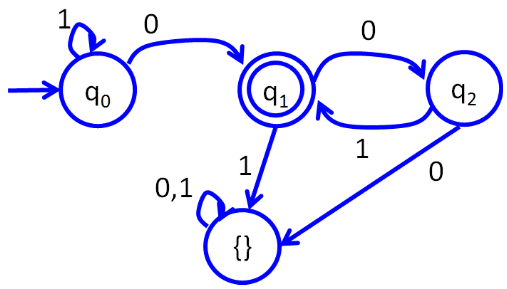
N11:



N12:



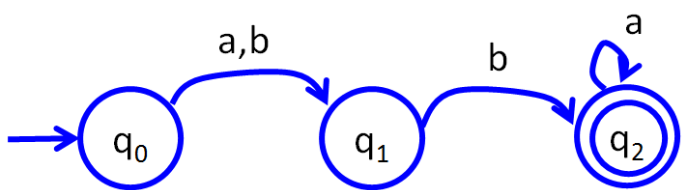
(1) **Enter {} state (exit all listed states for NFA)**
(2) **$1^*0(01)^*$**
(3)



5. For each regular expression using $\Sigma = \{a, b\}$:
(1) Provide three example words.
(2) Convert these regular expressions to a DFA or NFA

L13={$ab^*(ba)^*$}

L14={$(a \cup b)ba^*$}
(1) Examples: **ab, bb, aba, bba, abaaa, bbaa**



L15={$(bb)^* \cup (aa)^*$}

6. What is the minimum pumping length for each of these languages, showing these languages are regular? We use the alphabet $\Sigma = \{0,1,2\}$

L16={$00(0 \cup 1)^*12$}

L17={$0(22)^*10$}

L18={$111(202)^*210$}
p=9 smallest pumpable word: 111202210

If pumping length is p=5, how would you break up string w into x, y, and z for languages L below?

L19={ 20(11)*001 }, w=201111001
x=20 y=11 z=11001

L20={ (121)*001 } w=121001

7. Consider the language L21 ={01(101)*11}, what is the error in each of the following "pumping lemma" arguments?

Argument 1: Let us take w=0111, $w \in L21$. We cannot divide w=xyz such that $y^i z \in L21, i \geq 0$. For example, if x=0, y=11, and z=1, xy^2z = 011111$\notin L21$. Therefore, L21 is not regular.

Argument 2: Let us take w=0110110111, $w \in L21$. If we divide w=xyz as follows: x=0110110 , y=11, z=1, we cannot repeat y such that $xy^iz \in L21, i \geq 0$. For example, if xy^2z = 011011011111$\notin L21$. Therefore, L21 is not regular.

8. Prove these languages are not regular.

L24={$0^n1^{2n}0^{3n}$ | n>0 }

L25={1^{n^3} |n>0}

Proof by contradiction:

Assume L25 is regular. Consider w=1^{p^3} $w \in L25$ |w|>p

x=1^j y=1^k z=$1^{p^3-(j+k)}$

Try pumping: xy^2z -> 1^j 1^{2k} $1^{p^3-(j+k)}$ ->

Total number of 1's: j+k+p^3-(j+k) = p^3+k

Next word after 1^{p^3} will be $1^{(p+1)^3}$

Size of next-biggest word: $(p+1)^3$= (p^2+2p+1)(p+1) = p^3+3p^2+3p+1

$k \leq 0$, so p^3+k < p^3+3p^2+3p+1 p^3+k<$(p+1)^3$

Therefore, pumped w is <u>not</u> in L25.

9. For each of the following grammars, list three strings produced by the grammar

G26:

S -> AB | BA

A -> xAy | ε

B -> BzB | y

G27:

S -> A | AA

A -> 00 | 11

Examples: A -> **00**, A -> **11**, AA -> **0011**

G28:

A -> 11A00 | ε

10. Provide the languages described by two of the grammars:

G27 (from above)

~~(00U11)(00U11)*~~

00 U 11 U (00U11)(00U11)

G28 (from above)

10. Provide a grammar to produce the following languages

L32 = $\{0^n(11)^n \mid n \geq 0\}$
S -> 0S11 | ε

L33 = $\{01^*00^*\}$

L34 = $\{w \mid w = w^{Reverse}\}$ Examples: 00100, 10101, 1111

11. Convert the following grammars to Chomsky Normal Form

G29:
S -> xAy | BA
A -> z | AzA
B -> yB | ε

G30:
S -> BAB | ABA
A -> y | z
B -> x | AA | ε

G31:

S-> ByBy

B -> xBx | ε

<u>Answer corrected March 5, 10am</u>

Replace ε:

S -> ByBy | Byy | yBy | yy

B -> xBx | xx

Replace terminal--variable rules with all-variable rules:

S -> BU_yBU_y | BU_yU_y | U_yBU_y | U_yU_y

U_y -> y

B -> U_xBU_x | U_xU_x

U_x -> x

Replace 3^+ variable rules with 2-variable rules

S -> BC | BE | U_yD | U_yU_y

C->U_yD

D->BU_y

E->U_yU_y

U_y -> y

B -> U_xF | U_xU_x

F -> BU_x

U_x -> x

<u>Old answer with errors</u>

~~Replace ε:~~

~~S -> ByBy | ByB | BBy | BB~~

~~B -> xBx | xx~~

~~Replace terminal--variable rules with all-variable rules:~~

~~S -> BU_yBU_y | BBU_y | BU_yB | BB~~

~~U_y -> y~~

~~B -> U_xBU_x | $U_x U_x$~~

~~U_x -> x~~

~~Replace 3$^+$ variable rules with 2-variable rules~~

~~**S -> BC | BD | BE | BB**~~

~~**C -> U_yD**~~

~~**D -> BU_y**~~

~~**E -> U_yB**~~

~~**U_y -> y**~~

~~**B -> U_xF | $U_x U_x$**~~

~~**F -> BU_x**~~

~~**U_x -> x**~~