1. Consider the state diagram for the following DFAs. For each, answer the following questions:
(1) What state is reached by the input: w=00110?
(2) What is the transition function?
(3) What is the language recognized?

**M1:**

(1) $q_3$ 

(2)

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(3) \{w \mid w \text{ is 0 followed by odd number of digits or } w \text{ is 1 followed by even number of digits} \} = \{1((0 \cup 1)(0 \cup 1))^* \cup 0(0 \cup 1)((0 \cup 1)(0 \cup 1))^* \}

**M2:**
2. Define a machine to recognize the following languages in the alphabet \( \Sigma = \{1,2,3\} \)
(5 points)

- **L4**: \( \{w \mid \text{the product of input symbols is even}\} \)
  Examples: 111 (1x1x1=1 is odd - reject), 233 (2x3x3=18 is even - accept)

- **L5**: \( \{w \mid \text{numbers entered in non-decreasing order}\} \)
  Examples: 112223, 122333

- **L6**: \( \{w \mid \text{first two symbols are identical}\} \)
  Examples: 001213, 333212, 3310013

3. Prove the following languages are regular, using the alphabet \( \Sigma = \{a, b, c\} \):

- **L7**: \( \{w \mid \text{w contains an odd number of b's}\} \)

- **L8**: \( \{w \mid \text{w contains the sequence bcb}\} \) (Examples: aabbcbb or ccbcba)
L9={w | w does not have three a’s in a row}
Construct a DFA that recognizes three a’s in a row, then flip the accept and reject states.

4. Consider the following NFAs. For each, answer:
   (1) what state(s) will be reached by the input: 0011
   (2) provide a regular expression to describe the recognized language
   (3) For N11 and N12, convert NFA to DFA

N10:

N11:
(1) $q_2$  
$q_0 (0) \rightarrow q_1 (0) \rightarrow q_0 (1) \rightarrow q_1 (1) \rightarrow q_2$

(2) $(0U1)(0(0U1))^*1$ or alternatively: $\Sigma (0\Sigma)^* 1$

N12:

5. For each regular expression using $\Sigma = \{a, b\}$:
(1) Provide three example words.
(2) Convert these regular expressions to a DFA or NFA

$L13 = \{ab^*(ba)^*\}$
(1) a, ababa, abbb, abbbaba
(2)
6. What is the minimum pumping length for each of these languages, showing these languages are regular? We use the alphabet $\Sigma = \{0,1,2\}$

$L_{16} = \{00(0 \cup 1)^*12\}$
\[ p=5: \text{00012 or 00112 are the smallest strings you can pump}\]

$L_{17} = \{0(22)^*10\}$

$L_{18} = \{111(202)^*210\}$

If pumping length is $p=5$, how would you break up string $w$ into $x$, $y$, and $z$ for languages $L$ below?

$L_{19} = \{20(11)^*001\}, \quad w=20111001$

$L_{20} = \{(121)^*001\} \quad w=121001$
\[ x=\epsilon, \quad y=121, \quad z=001\]

7. Consider the language $L_{21} = \{01(101)^*11\}$, what is the error in each of the following “pumping lemma” arguments?

**Argument 1:** Let us take $w=0111, \quad w \in L_{21}$. We cannot divide $w=xyz$ such that $xy^iz \in L_{21}, i \geq 0$. For example, if $x=0, \quad y=11, \quad$ and $z=1, \quad xy^2z = 011111\notin L_{21}$. Therefore, $L_{21}$ is not regular.
Argument 2: Let us take \( w=0110110111, w \in L_{21} \). If we divide \( w=xyz \) as follows: 
\( x=0110110, y=11, z=1 \), we cannot repeat \( y \) such that \( xy^iz \in L_{21}, i \geq 0 \). For example, if \( xy^2z = 011011011111 \notin L_{21} \). Therefore, \( L_{21} \) is not regular.

We divided \( w \) improperly to allow pumping. There exist other divisions of \( w \) that are pumpable, such as: \( x=01, y=101, z=10111 \)

8. Prove these languages are not regular.

\( L_{24} = \{0^n1^{2n}0^{3n} | n>0 \} \)

\( L_{25} = \{1^n^3 | n>0 \} \)

9. For each of the following grammars, list three strings produced by the grammar

G26:
\[
S \rightarrow AB \mid BA \\
A \rightarrow xAy \mid \varepsilon \\
B \rightarrow BzB \mid y
\]

G27:
\[
S \rightarrow A \mid AA \\
A \rightarrow 00 \mid 11
\]
G28:
A -> 11A00 | ε

Examples: ε, 1100, 11110000, 111111000000

10. Provide the languages described by two of the grammars:

G27 (from above)

G28 (from above)

L={(11)^n(00)^n | n ≥ 0}

10. Provide a grammar to produce the following languages

L32 = {0^n(11)^n | n≥0}

L33 = {01*00*}
Think of a standard DFA and convert from there
G33:
S -> 0R₁
R₁ -> 1R₁ | 0R₂
R₂ -> 0R₂ | ε

L34 = {w | w= w_{Reverse}}    Examples: 00100, 10101, 1111
11. Convert the following grammars to Chomsky Normal Form

**G29:**
S \rightarrow xAy \mid BA  
A \rightarrow z \mid AzA  
B \rightarrow yB \mid \varepsilon

**G30:**
S \rightarrow BAB \mid ABA  
A \rightarrow y \mid z  
B \rightarrow x \mid AA \mid \varepsilon

Remove \varepsilon
S \rightarrow BAB \mid ABA \mid AB \mid BA \mid A \mid AA  
A \rightarrow y \mid z  
B \rightarrow x \mid AA

Remove unit S \rightarrow A
S \rightarrow BAB \mid ABA \mid AB \mid BA \mid y \mid z \mid AA  
A \rightarrow y \mid z  
B \rightarrow x \mid AA

Convert 3+ variable rules to chain of 2-variable rules
S \rightarrow BC \mid AD \mid AB \mid BA \mid y \mid z \mid AA  
C \rightarrow AB  
D \rightarrow BA  
A \rightarrow y \mid z  
B \rightarrow x \mid AA

**G31:**
S \rightarrow ByBy  
B \rightarrow xBx \mid \varepsilon