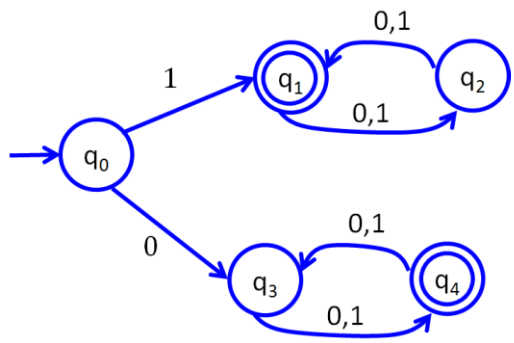
1. Consider the state diagram for the following DFAs. For each, answer the following questions:
(1) What state is reached by the input: w=00110 ?
(2) What is the transition function?
(3) What is the language recognized?

M1:



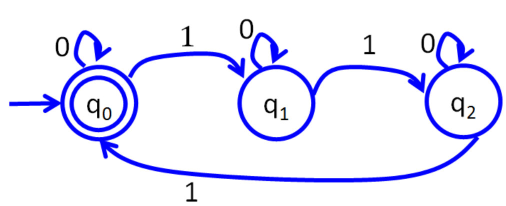
(1) **q_3** (q_0 (0) -> q_3 (0) -> q_4 (1) -> q_3 (1) -> q_4 (0) -> q_3)

(2)

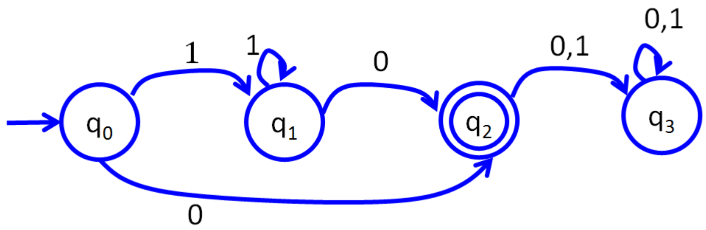
	0	1
q_0	q_3	q_1
q_1	q_2	q_2
q_2	q_1	q_1
q_3	q_4	q_4
q_4	q_3	q_3

(3) {w | w is 0 followed by odd number of digits or w is 1 followed by even number of digits} $\{1\big((0 \cup 1)(0 \cup 1)\big)^* \cup 0(0 \cup 1)\big((0 \cup 1)(0 \cup 1)\big)^*\}$

M2:



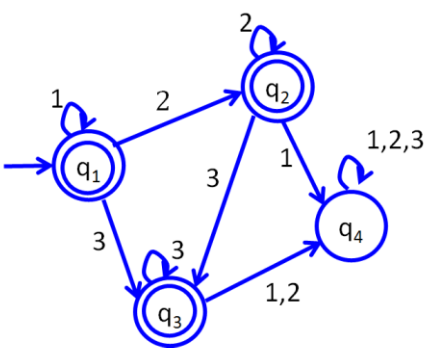
M3:



2. Define a machine to recognize the following languages in the alphabet
$\Sigma = \{1,2,3\}$
(5 points)

L4={w | the product of input symbols is even} E.g., 111->1x1x1=1 is odd-reject,
233 -> 2x3x3 = 18 is even-accept

L5={w | numbers entered in non-decreasing order} Examples: 112223, 122333



L6={w | first two symbols are identical} Examples: 001213, 333212, 3310013

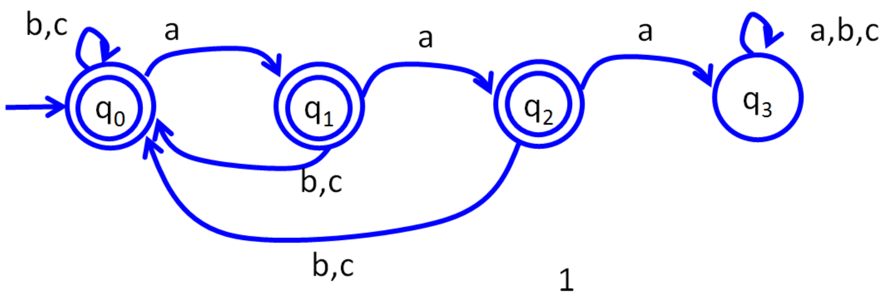
3. Prove the following languages are regular, using the alphabet $\Sigma = \{a, b, c\}$:

L7={w | w contains an odd number of b's}

L8={w | w contains the sequence bcb} (Examples: aabbbcbb or ccbcba)

L9={w | w does not have three a's in a row}

Construct a DFA that recognizes three a's in a row, then flip the accept and reject states.



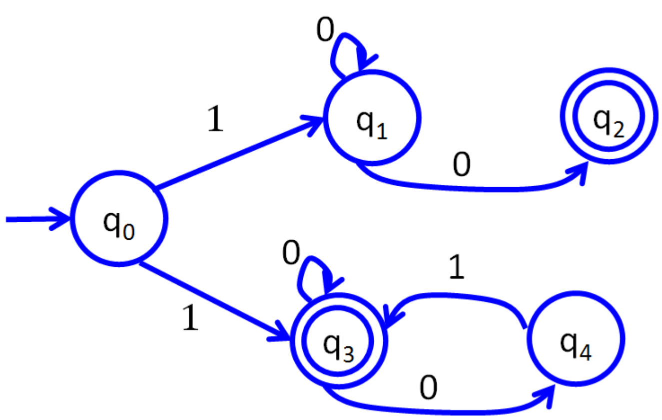
4. Consider the following NFAs. For each, answer:

(1) what state(s) will be reached by the input: 0011

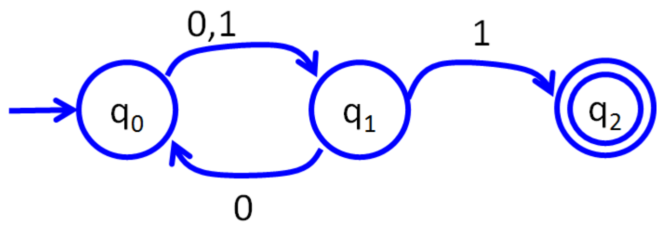
(2) provide a regular expression to describe the recognized language

(3) **For N11 and N12**, convert NFA to DFA

N10:



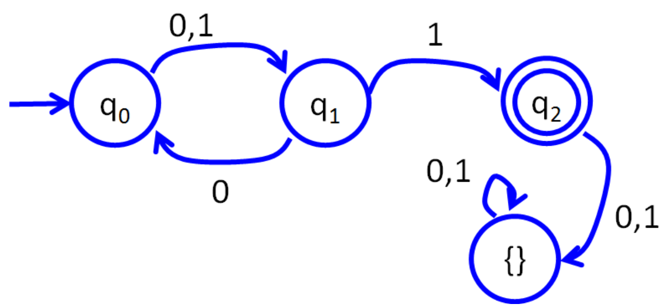
N11:



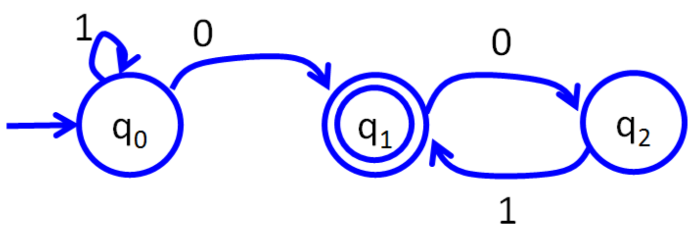
(1) $\mathbf{q_2}$ q_0 (0)-> q_1 (0) -> q_0 (1) -> q_1 (1) -> q_2

(2) $\mathbf{(0U1)(0(0U1))^*1}$ or alternatively: $\mathbf{\Sigma(0\Sigma)^*1}$

(3)



N12:



5. For each regular expression using $\Sigma = \{a, b\}$:

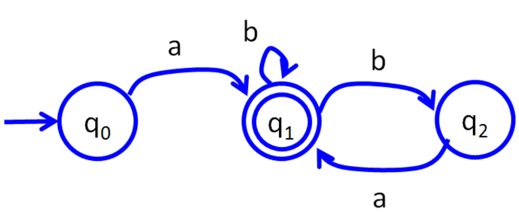
(1) Provide three example words.

(2) Convert these regular expressions to a DFA or NFA

L13=$\{ab^*(ba)^*\}$

(1) **a, ababa, abbb, abbbaba**

(2)



$L14=\{(a \cup b)ba^*\}$

$L15=\{(bb)^* \cup (aa)^*\}$

6. What is the minimum pumping length for each of these languages, showing these languages are regular? We use the alphabet $\Sigma = \{0,1,2\}$

$L16=\{00(0 \cup 1)^*12\}$

p=5: 00012 or 00112 are the smallest strings you can pump

$L17=\{0(22)^*10\}$

$L18=\{111(202)^*210\}$

If pumping length is p=5, how would you break up string w into x, y, and z for languages L below?

$L19=\{ 20(11)^*001 \}$, w=201111001

$L20=\{ (121)^*001 \}$ w=121001

x=ε y=121 z=001

7. Consider the language $L21 =\{01(101)^*11\}$, what is the error in each of the following "pumping lemma" arguments?

Argument 1: Let us take w=0111, $w \in L21$. We cannot divide w=xyz such that $xy^iz \in L21, i \geq 0$. For example, if x=0, y=11, and z=1, xy^2z = 011111$\notin L21$. Therefore, L21 is not regular.

Argument 2: Let us take w=0110110111, $w \in L21$. If we divide w=xyz as follows: x=0110110 , y=11, z=1, we cannot repeat y such that $xy^iz \in L21, i \geq 0$. For example, if xy^2z = $011011011111 \notin L21$. Therefore, L21 is not regular.

We divided w improperly to allow pumping. There exist other divisions of w that <u>are</u> pumpable, such as: x=01, y=101, z=10111

8. Prove these languages are not regular.

L24=$\{0^n1^{2n}0^{3n} \mid n>0\}$

L25=$\{1^{n^3} \mid n>0\}$

9. For each of the following grammars, list three strings produced by the grammar

G26:
S -> AB | BA
A -> xAy | ε
B -> BzB | y

G27:
S -> A | AA
A -> 00 | 11

G28:

A -> 11A00 | ε

Examples: ε, 1100, 11110000, 111111000000

10. Provide the languages described by two of the grammars:

G27 (from above)

G28 (from above)

L=$\{(11)^n(00)^n \mid n \geq 0\}$

10. Provide a grammar to produce the following languages

L32 = $\{0^n(11)^n \mid n \geq 0\}$

L33 = $\{01^*00^*\}$

Think of a standard DFA and convert from there

G33:

S -> $0R_1$

R_1 -> $1R_1$ | $0R_2$

R_2 -> $0R_2$ | ε

L34 = $\{w \mid w=w^{Reverse}\}$ **Examples: 00100, 10101, 1111**

11. Convert the following grammars to Chomsky Normal Form

G29:
S -> xAy | BA
A -> z | AzA
B -> yB | ε

G30:
S -> BAB | ABA
A -> y | z
B -> x | AA | ε

Remove ε
S -> BAB | ABA | AB | BA | A | AA
A -> y | z
B -> x | AA

Remove unit S->A
S -> BAB | ABA | AB | BA | y | z | AA
A -> y | z
B -> x | AA

Convert 3^{+} variable rules to chain of 2-variable rules
S->BC | AD | AB | BA | y | z | AA
C -> AB
D -> BA
A -> y | z
B -> x | AA

G31:
S-> ByBy
B -> xBx | ε