Consider a classifier hypothesis set of squares. A single hypothesis \( h \) is a square with a fixed size and location. Four example hypotheses are shown.

And here is examples of \( h \) that will help shatter a set of three data points.

For each data set:
- What is a set of 4 shatterable points (“none” is a possible answer)
- What is the VC dimension?

Example 1:

Example 2:

4 points: None
VC: 3 dims (e.g., B,C,E)
Example 3:

Consider the following HMM. It uses a thermometer to attempt to predict the weather.

We begin with the following estimate for our HMM parameters:
\[ \Pi_{\text{snow}} = 0.2 \quad \Pi_{\text{rain}} = 0.3 \quad \Pi_{\text{sunny}} = 0.3 \quad \Pi_{\text{cloudy}} = 0.2 \]

(We COULD actually learn a Gaussian function for the temperature for each state. Here, we’ll just do a discrete probability table.)

We receive a new sequence of temperatures and wish to update our HMM parameters.
Sequence:
Cold  Cold  Hot  Mild  Hot

Correct alpha values are in black. Made-up alpha values are in color parentheses. You will have to find the real values below. You can use the made-up value in calculating $S_t$ values further below.

\[
\alpha_t(i)
\]

<table>
<thead>
<tr>
<th></th>
<th>t:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snow</td>
<td>?? (.11)</td>
<td>.08</td>
<td>0</td>
<td>.00011</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Rain</td>
<td>0.15</td>
<td>?? (.04)</td>
<td>.0082</td>
<td>.0017</td>
<td>.00049</td>
<td></td>
</tr>
<tr>
<td>Sunny</td>
<td>?? (.08)</td>
<td>0</td>
<td>.0056</td>
<td>?? (.0033)</td>
<td>.0020</td>
<td></td>
</tr>
<tr>
<td>Cloudy</td>
<td>0.04</td>
<td>.027</td>
<td>?? (.0044)</td>
<td>.0053</td>
<td>.00030</td>
<td></td>
</tr>
</tbody>
</table>

Correct beta values are in black. Made-up beta values are in color parentheses. You will have to find the real values below. You can use the made-up value in calculating $S_t$ values further below.

\[
\beta_t(i)
\]

<table>
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<tr>
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<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snow</td>
<td>.0067</td>
<td>.0062</td>
<td>.13</td>
<td>.05</td>
<td></td>
</tr>
<tr>
<td>Rain</td>
<td>.0097</td>
<td>?? (.011)</td>
<td>.13</td>
<td>?? (.08)</td>
<td></td>
</tr>
<tr>
<td>Sunny</td>
<td>.0028</td>
<td>.087</td>
<td>?? (.11)</td>
<td>.52</td>
<td></td>
</tr>
<tr>
<td>Cloudy</td>
<td>.0062</td>
<td>.047</td>
<td>.121</td>
<td>?? (.11)</td>
<td></td>
</tr>
</tbody>
</table>

Find the missing values in the tables above.

\[
\alpha_{t=2}(Rain): 0.05
\]

CORRECTED Dec 12:

\[
\alpha_{t=4}(Sunny): 0.3 \times (0 + 0 + 0.0056 \times 0.7 + 0.0044 \times 3) = 0.3 \times (0.00392 + 0.00132) = 0.0016
\]

From: Snow, Rain, Sunny, Cloudy

\[
0.0017
\]
CORRECTED Dec 12:
\[ \beta_{t=3}(Sunny): 0 + 0 + .52 \times 3 \times .7 + .11 \times 7 \times 3 = .109 + .0231 = .13 \quad 0.17 \]
To: Snow, Rain, Sunny, Cloudy

What are the values:

\[ S_2(\text{cloudy}) \]

\[ S_3(\text{snow,sunny})= \]

CORRECTED DEC 12:
\[ S_1(\text{rain}): \frac{.0097 \times .15}{.001455 + .000737 + .00024 + .000248} = 0.543 \]

Now let us presume the following S values (these are made-up values):

\[ S_t(i) \]

<table>
<thead>
<tr>
<th></th>
<th>t</th>
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<th>2</th>
<th>3</th>
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<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snow</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Rain</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td>0.3</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Sunny</td>
<td>0.1</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>Cloudy</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>

\[ S_t(i,j) \]

<table>
<thead>
<tr>
<th></th>
<th>t</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rain, Cloudy</td>
<td>.1</td>
<td>.4</td>
<td>.3</td>
<td>.2</td>
<td></td>
</tr>
<tr>
<td>Sunny, Rain</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

What are the resulting estimate of the following parameters?
\[ \Pi_{\text{rain}} \]
\[ \Pi_{\text{cloudy}} = S_1(\text{cloudy}) = 0.1 \]

\[ A_{\text{rain,cloudy}} = \frac{\sum_{t} S_t(\text{rain,cloudy})}{\sum_{t} S_t(\text{cloudy})} = \frac{0.1 + 0.4 + 0.3 + 0.2}{0.5 + 0.4 + 0.3 + 0.3} = \frac{1}{1.5} = 0.67 \]

\[ A_{\text{sunny,rain}} \]

\[ \phi_{\text{hot,rain}} \quad \phi_{\text{mild,sunny}} \]