

Calculus rules

$$\frac{d}{dx} x^a = ax^{a-1} \quad \frac{d}{dx} kf(x) = kf'(x)$$

$$\frac{d}{dx} [f(x) + g(x)] = f'(x) + g'(x)$$

$$\frac{d}{dx} (f(g(x))) = g'(x) \cdot f'(g(x))$$

$$\frac{d}{dx} \log x = \frac{1}{x} \quad \frac{d}{dx} e^x = e^x \quad \frac{d}{dx} f(w) = 0$$

Log rules

$$\log ab = \log a + \log b$$

$$\log a^n = n \log a \quad \log e^a = \log(\exp(a)) = a$$

Update rules

Basic logistic update:

$$w_j \leftarrow w_j + \epsilon x_j^i (y^i - g(\mathbf{w}^T \mathbf{x}))$$

SVM optimization with slack:

$$\operatorname{argmin}_{\mathbf{w}} \mathbf{w}^T \mathbf{w} + C \sum_i \xi^i$$

such that

$$\mathbf{w}^T \mathbf{x}^+ + b \geq +1 - \xi^i$$

$$\mathbf{w}^T \mathbf{x}^- + b \leq -1 + \xi^i$$

$$\mathbf{w} = \sum_i \alpha^i y^i \mathbf{x}^i$$

For PCA: $z_q^i = (\mathbf{u}^q)^T \mathbf{x}^i$ (Linear algebra version: $\mathbf{z} = \mathbf{U}^T \mathbf{x}^i$)

For Neural Network:

$$\delta_k^{top,i} = (1 - r_k^{top,i})(y^i - r_k^{top,i})r_k^{top,i}$$

(Linear algebra version: $\boldsymbol{\delta}^{top} = (\mathbf{r}^{top,i})^T ((1 - \mathbf{r}^{top,i}) \circ (\mathbf{y}^i - \mathbf{r}^{top,i}))$)

HMM

$$\beta_t(i) = \sum_j A_{j,i} \phi_{o_{t+1},j} \beta_{t+1}(j)$$

$$S_t(i,j) = \frac{\alpha_t(j) A_{i,j} \phi_{o_{t+1},i} \beta_{t+1}(i)}{\sum_{f,g} \alpha_t(g) A_{f,g} \phi_{o_{t+1},f} \beta_{t+1}(f)}$$

Linear algebra for $\boldsymbol{\beta}_t$

$$\boldsymbol{\beta}_t = \mathbf{A}^T (\boldsymbol{\phi}_{o_{t+1}} \circ \boldsymbol{\beta}_{t+1})$$

From M-step: $A_{i,j} = \frac{\sum_t S_t(i,j)}{\sum_k \sum_t S_t(k,j)}$