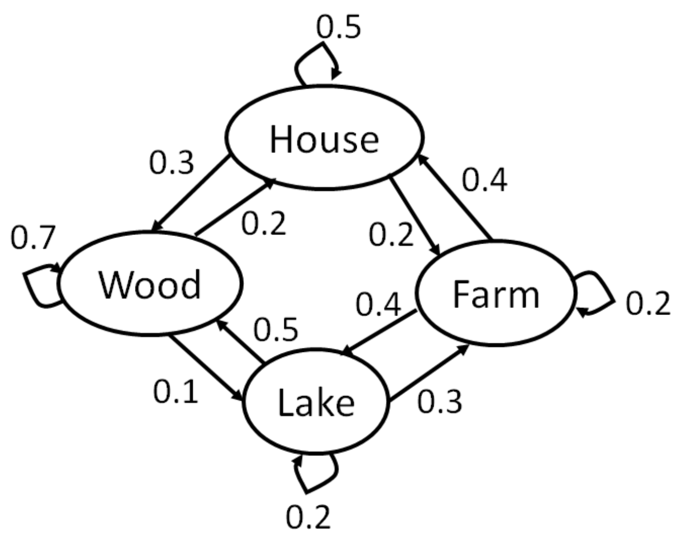
Fake Homework

Presume the following Markov Model



$\Pi_{House} = 0.4$

$\Pi_{Farm} = 0.2$

$\Pi_{Woods} = 0.3$

$\Pi_{Lake} = 0.1$

1. What is the probability of each of the following state sequences?

   (a) Farm, House, Farm, Lake

   (b) Woods, Woods, Farm, House, Farm

   (c) Farm, Farm, House

Let us expand the above model to be a full HMM using the emission probabilities below:
$\boldsymbol{\phi_{i,j} = P(o_t = x_i | q_t = s_j)}$:

q\o	quack	woof	television	roar	bah	speech
House (locat 1)	0.1	0.2	0.3	0	0.1	0.3
Farm (locat 2)	0.3	0.2	0	0	0.4	0.1
Woods (locat 3)	0.1	0.3	0	0.6	0	0
Lake (locat 4)	0.7	0.1	0	0	0	0.2

(For reference, you can presume a duck quacks, a dog woofs, a bear roars, a sheep bahs, and a human speaks.)

2. What is the probability of each of the following sequences of states and observations:

   (a) P(q_1=Woods, o_1=woof, q_2=House, o_2=bah)

(b) P(q_1=House, o_1=woof, q_2=Farm, o_2=speech)

(c) P(q_1=Woods, o_1=roar, q_2=Woods, o_2=quack)

3. Suppose we observe the following sounds in order:

o_1=woof, o_2=roar, o_3=quack

Given the observations above:

(a) Use the Viterbi algorithm to assess the most likely set of states.

As you work on this problem, provide the values for

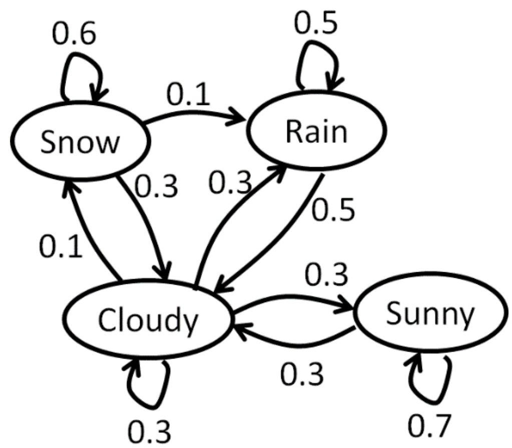
(b) $\delta_1(Farm)$
(c) $\delta_2(Woods)$

4. Consider the following HMM. It uses a thermometer to attempt to predict the weather.

We begin with the following estimate for our HMM parameters:
$\Pi_{snow} = 0.2$ $\Pi_{rain} = 0.3$ $\Pi_{sunny} = 0.3$ $\Pi_{cloudy} = 0.2$

$\phi_{o,i}$:

	Cold	Mild	Hot
Snow	0.8	0.2	0
Rain	0.5	0.3	0.2
Sunny	0	0.3	0.7
Cloudy	0.2	0.7	0.1



(We COULD actually learn a Gaussian function for the temperature for each state. Here, we'll just do a discrete probability table.)

We receive a new sequence of temperatures and wish to update our HMM parameters.

Sequence:
Cold Cold Hot Mild Hot

Correct alpha values are in black. Made-up alpha/beta values are in parentheses with ?? before. You will have to find some of the real values below. You should use the made-up value in calculating S_t values further below.

$\alpha_t(i)$

t:	1	2	3	4	5
Snow	?? (.11)	.08	0	.00011	0
Rain	0.15	?? (.04)	.0082	.0017	.00049
Sunny	?? (.08)	0	.0056	?? (.0033)	.0020
Cloudy	0.04	.027	?? (.0044)	.0053	.00030

$\beta_t(i)$

t:	1	2	3	4
Snow	.0067	.0062	.13	.05
Rain	.0097	?? (.011)	.13	?? (.08)
Sunny	.0028	.087	?? (.11)	.52
Cloudy	.0062	.047	.121	?? (.11)

a) Find the following missing values in the tables above.

$\alpha_1(Sunny)$ $\alpha_3(Cloudy)$ $\alpha_4(Sunny)$

$\beta_2(Rain)$ $\beta_3(Sunny)$ $\beta_4(Cloudy)$

b) What are the values:

S_2(cloudy) S_3(snow,sunny) S_1(rain)

Now let us presume the following S values (these are made-up values):

$S_t(i)$

t	1	2	3	4	5
Snow	0.3	0.3	0.1	0.2	0.1
Rain	0.5	0.4	0.3	0.3	0.2
Sunny	0.1	0.1	0.3	0.1	0.4
Cloudy	0.1	0.2	0.3	0.4	0.3

$S_t(i,j)$

t	1	2	3	4
Rain, Cloudy	.1	.4	.3	.2
Sunny, Rain	0	0	0	0

c) What are the values below?

Π_{rain} Π_{cloudy}

$A_{rain,cloudy}$ $\phi_{mild,sunny}$